Underlay Cognitive Networks with Partial Relay Selection and Primary-User Interference

Diana Pamela Moya Osorio and Edgar Eduardo Benitez Olivo

Abstract—Most related works addressing cognitive relaying networks focus on the impact of the interference constraint imposed by the primary receivers on the performance of the secondary network, while neglecting the interference coming from the primary transmitters. In this paper, we assess the outage performance of a cognitive relaying network composed by a source communicating with a destination under the assistance of one relay, which is selected from a cluster of decode-and-forward relays, using partial relay selection. Exact and asymptotic closed-form expressions for the outage probability of the considered network are derived over Rayleigh fading channels, which are validated through Monte Carlo simulations.

Keywords—cognitive radio, decode-and-forward, interference, outage probability, partial relaying selection.

I. INTRODUCTION

Spectrum sharing in cognitive radio networks has arisen as a promising solution to improve the spectrum-usage efficiency of emerging fifth generation (5G) wireless communication systems, by allowing unlicensed (secondary) users to share the same frequency band owned by licensed (primary) users [1], [2]. One common approach to make this possible is the underlay spectrum-sharing technique [3], whereby a secondary node is enabled to access the licensed frequency band, as long as its transmit power is carefully controlled, so as to constrain the resulting interference on the primary receiver to a limit, referred to as interference temperature [2].

On the other hand, in order to improve the transmission reliability, extend the coverage area, and increase the system throughput—without the requirement of additional transmit power or multiple antennas at the nodes—cooperative relaying techniques have been proposed [4], [5]. The main concept behind cooperative communications consists in leveraging the antennas of neighboring nodes in the network, in order to relay the signals of each other, thus emulating a spatially-distributed antenna array. According to the relaying protocol, cooperative nodes can be categorized as (i) decode-and-forward (DF) relays, which decode and re-encode the information signal before forwarding it, or (ii) amplify-and-forward (AF) relays, which forward the information signal without hard decoding [5].

In the context of cooperative dual-hop relaying networks with multiple relays, relay selection techniques have been considered because of their efficient use of system resources (such as power and bandwidth), since only two orthogonal channels are required, regardless of the number of relays [6], [7]. Two well-know relay selection techniques are opportunistic relay selection and partial relay selection. While the former is performed considering the channel state information (CSI) of the entire network, that is, the CSI of all the source-relay (first-hop) links and relay-destination (second-hop) links [6], the latter has the advantage of requiring CSI of only the first-hop links, or alternatively, only the second-hop links, thus alleviating the feedback overhead [7].

The combination of underlay spectrum sharing and cooperative relaying in cognitive relaying networks (CRN) can potentially improve the system performance even further. Indeed, an inordinate amount of attention has been focused on CRNs (see, for example, [8], [9] and the references therein). However, an assumption widely considered in the related literature is that the effect of the interference from the primary users on the CRN performance is negligible. In fact, this can be possible if the primary source is located far away from the secondary nodes, so that the interference links from the primary transmitter to the secondary receivers are subject to a severe attenuation due to the path loss and shadowing. On the other hand, because primary and secondary users share the same frequency band, a mutual interference between them could be inevitable for certain scenarios.

Few works have examined the impact of the primary-user interference on the performance of CRNs [10]–[12]. Among them, in [10], the effect of the interference inflicted by multiple primary sources on the outage performance of a cognitive DF relaying network was analyzed. However, in that work, the analysis was performed by considering only an interference-limited scenario. In [11], the effect of the primary user interference and outdated CSI on the outage probability of a three-node CRN, with multiple-antenna secondary nodes and DF relaying, was examined. In [12], the outage performance of a spectrum sharing DF generalized order relay selection network in the presence of interference from primary user was studied. However, in [10] and [12], the transmit power of secondary nodes was considered to be unlimited, since it is defined as the ratio between the interference temperature and the CSI of the interference link from the secondary source to the primary receiver, whereas in [11], the transmit power of secondary nodes does not consider the CSI of the referred interference link.

In order to fill in part this gap, in this paper we perform an exact analysis in terms of the outage probability for a CRN consisting of a source and a destination, which communicate with the assistance of one relay, selected from a cluster of DF relays. Motivated by the feedback-overhead efficiency, we...
undergo independent and identically distributed (i.i.d.) block Rayleigh fading and additive white Gaussian noise (AWGN) with mean power $N_0$. In this sense, the corresponding channel gains $g_{XY}=|h_{XY}|^2$, with $X \in \{S, R_n, A\}$ and $Y \in \{R_n, D, B\}$, are exponentially distributed random variables with mean values $\Omega_{XY}=E[|h_{XY}|^2]$, where it is assumed that the channel coefficients remain constant during the transmission of a data block, but change independently over the consecutive blocks. Considering this, $\gamma_{SR_n}=g_{SR}P_S/N_0$, $\gamma_{RN,D}=g_{RN,D}P_R/N_0$, $\gamma_{SB}=g_{SB}P_S/N_0$, $\gamma_{RB}=g_{RB}P_R/N_0$, $\gamma_{AB}=g_{AB}P_A/N_0$, and $\gamma_{AD}=g_{AD}P_A/N_0$ are the instantaneous received signal-to-noise ratios (SNRs) for the links $S \rightarrow R_n$, $R_n \rightarrow D$, $S \rightarrow B$, $R_n \rightarrow B$, $A \rightarrow R_n$, and $A \rightarrow D$, respectively. In these expressions, $P_S$, $P_R$, and $P_A$ are the corresponding transmit powers at $S$, $R_n$, and $A$. Then, by considering the constraints for the transmitted powers at $S$ and $R$, and by denoting $\gamma_P=P_A/N_0$ as the maximum transmit SNR at $A$, $\gamma_P=P_S/N_0=P_R/N_0$ as the maximum transmit SNR at $S$ and $R_n$, and $\bar{\gamma}_I=I/N_0$ as the maximum interference-to-noise ratio tolerated at $B$, we have that

$$\gamma_{SR_n} = \min \left\{ \frac{\gamma_I g_{SR_n}}{g_{SB}}, \frac{g_{SR_n}}{\gamma_P g_{SR_n}}, \frac{g_{SR_n}}{\gamma_P} \right\} \quad \gamma_{SR_n} = \begin{cases} \frac{\gamma_I g_{SR_n}}{g_{SB}}, & g_{SB} > \frac{\gamma_I}{\gamma_P} \\ \frac{g_{SR_n}}{\gamma_P g_{SR_n}}, & g_{SB} \leq \frac{\gamma_I}{\gamma_P} \end{cases}, \quad (1)$$

$$\gamma_{RN,D} = \min \left\{ \frac{\gamma_I g_{RN,D}}{g_{RB}}, \frac{g_{RB}}{\gamma_P g_{RN,D}}, \frac{g_{RB}}{\gamma_P} \right\} \quad \gamma_{RN,D} = \begin{cases} \frac{\gamma_I g_{RN,D}}{g_{RB}}, & g_{RB} > \frac{\gamma_I}{\gamma_P} \\ \frac{g_{RB}}{\gamma_P g_{RN,D}}, & g_{RB} \leq \frac{\gamma_I}{\gamma_P} \end{cases}. \quad (2)$$

Then, the instantaneous received signal-to-interference-plus-noise ratios (SNRs) at $R_n$ and $D$ are given, respectively, by

$$\gamma_{R_n} = \frac{\gamma_{SR_n}}{\gamma_{AR_n} + 1}, \quad (3)$$

$$\gamma_D = \frac{\gamma_{RN,D}}{\gamma_{AD} + 1}. \quad (4)$$

For the proposed system, one out of the $N$ relays is selected to cooperate on the communication process between $S$ and $D$, according to a partial relay selection criterion, which is based on the channel conditions of the links $S \rightarrow R_n$, $n = 1, \ldots, N$, (i.e., the first hops of the relaying links) and is given by

$$n^* = \arg\max_n \{\gamma_{R_n}\}. \quad (5)$$

### III. OUTAGE PROBABILITY

In this section, the performance of the proposed system is analyzed in terms of the outage probability. By definition, the secondary network is in outage when the instantaneous received end-to-end SNR, $\gamma_{c2c}$, drops below a certain threshold $\tau \triangleq 2^{2\bar{\gamma}_I} - 1$, where $\bar{\gamma}_I$ is the target spectral efficiency given in bits/s/Hz. Thus, the outage probability can be formulated as in the following theorem.

**Theorem 1:** the outage probability of an underlay cognitive relaying network with partial relay selection, which is subject to primary-user interference, is given by

$$P_{\text{OUT}} = P_{\gamma_D}(\tau) - P_{\gamma_{R_n}}(\tau)^N [P_{\gamma_D}(\tau) - 1], \quad (6)$$
where $F_{\gamma_{R_a}}(\tau)$ is given by (7), shown at the top of the next page. Moreover, $F_{\gamma_{I}}(\tau)$ can also be expressed as in (7), by replacing $\Omega_{SB}$, $\Omega_{AR}$, and $\Omega_{SR}$ by $\Omega_{RB}$, $\Omega_{AD}$, and $\Omega_{RD}$, in this order.

**Proof:** according to the definition of outage probability, it follows that

\[
P_{\text{OUT}} = \Pr (\gamma_{2e} < \tau) = \Pr (\min \{\gamma_{R_a}, \gamma_{I}\} < \tau|n^* = n) \Pr (n^* = n) = \sum_{i=0}^{N} \Pr (\min \{\gamma_{R_a}, \gamma_{I}\} < \tau) \Pr (\gamma_{R_a} > \max \{\gamma_{R_i}\})
\]

\[
= T_1 + T_2 + T_3,
\]

(9)

where the step (a) is obtained by applying the Total Probability Theorem [14]. Besides, the terms $T_1$, $T_2$, and $T_3$ are obtained by splitting the minimum function into three regions, as follows: i) $\gamma_{R_a} < \tau$ and $\gamma_{I} > \tau$, ii) $\gamma_{R_a} < \tau$ and $\gamma_{I} < \tau$, and iii) $\gamma_{R_a} > \tau$ and $\gamma_{I} < \tau$. Thereby, $T_1$ can be derived as

\[
T_1 = \sum_{i=0}^{N} \Pr (\gamma_{R_a} < \tau, \gamma_{I} > \tau) \Pr (\gamma_{R_a} > \max \{\gamma_{R_i}\})
\]

\[
= \sum_{i=0}^{N} \int_{0}^{\tau} F_{\gamma_{R_a}}(\phi)^{N-1} [1 - F_{\gamma_{I}}(\phi)] f_{\gamma_{R_a}}(\phi) d\phi
\]

\[
= F_{\gamma_{R_a}}(\tau)^{N} [1 - F_{\gamma_{I}}(\tau)].
\]

(10)

By applying the same rationale, $T_2$ and $T_3$ can be derived as

\[
T_2 = F_{\gamma_{R_a}}(\tau)^{N} F_{\gamma_{I}}(\tau),
\]

(11)

\[
T_3 = F_{\gamma_{I}}(\tau) \left[1 - F_{\gamma_{R_a}}(\tau)^{N}\right].
\]

(12)

In the above expressions, the CDFs $F_{\gamma_{R_a}}(\tau)$ and $F_{\gamma_{I}}(\tau)$ can be obtained as

\[
F_{\gamma_{R_a}}(\tau) = \Pr (\gamma_{R_a} < \tau|g_{SB}) = \Pr \left(\frac{\gamma S_{R_a}}{\gamma A R_{a} + 1} < \tau|g_{SB}\right)
\]

\[
= \int_{0}^{\infty} \Pr (\gamma S_{R_a} < \tau(\omega + 1)|g_{SB}) f_{\gamma_{R_a}}(\omega) d\omega
\]

\[
= I_1 + I_2,
\]

(13)

where, in light of (1), the terms $I_1$ and $I_2$ can be expressed as

\[
I_1 = \int_{\gamma_{1}/\gamma_{P}}^{\gamma_{1}/\gamma_{P}} \int_{0}^{\infty} F_{g_{SR}} \left[\frac{(\omega + 1)}{\gamma_{P}}\right] f_{\gamma_{R_a}}(\omega) f_{g_{SB}}(w) d\omega dw
\]

\[
I_2 = \int_{0}^{\gamma_{1}/\gamma_{P}} \int_{0}^{\infty} F_{g_{SR}} \left[\frac{(\omega + 1)}{\gamma_{P}}\right] f_{\gamma_{R_a}}(\omega) f_{g_{SB}}(w) d\omega dw.
\]

(14)

(15)

Accordingly, by solving exponential-functions integrals and considering [15, eq. (3,352-2)], the terms $I_1$ and $I_2$ can be obtained in closed form, as in the first and second lines in (7), respectively. An identical analysis can be followed to obtain $F_{\gamma_{I}}(\tau)$ by considering (2). Therefore, an exact closed-form expression can be determined for the outage probability in (6), by summing up the terms $T_1$, $T_2$, and $T_3$, according to (9).

In the following theorem, a closed-form asymptotic expression at high SNR is introduced.

**Theorem 2:** the asymptotic outage probability of an underlay cognitive relaying network with partial relay selection, which is subject to primary-user interference, is given by (8), shown at the next page.

**Proof:** By considering that the maximum-available transmit powers at the secondary terminals, $P_S$ and $P_R$, vary independently of the maximum interference power tolerated at the primary destination, $I$, it follows that all the terms proportional to $1/\gamma_{P}$ tend to zero as $P_S$ increases at high SNR.

In addition, using the Maclaurin expression for the exponential function [15, Eq. (0.318.2)], we have that $e^{-x} \simeq 1 - x$ for small $x$. Applying this into (6), and neglecting the high-order terms with regard to $1/\gamma_p$, by some mathematical manipulations, a asymptotic closed-form expression for the outage probability can be obtained as in (8).

**Remark 1:** From (8), we can notice that the asymptotic outage probability is independent of $\gamma_p$, thus resulting in a floor. As a consequence, the diversity order [17] for the considered system turns out to be zero.

IV. Numerical Results and Discussions

In this section, our analytical expressions are validated through illustrative sample cases via Monte Carlo simulations. For this purpose, we assume a bi-dimensional network topology, where $S$ is located at the coordinates $(0; 0)$, the $N$ relays are clustered together and co-located at $(0.5; 0)$, $D$ is located at $(1; 0)$, and $A$ and $B$ are located at $(0; 1)$ and $(1; 1)$, respectively. Furthermore, we consider that the average channel gains of all links are determined by the path-loss, i.e., $\Omega_{XY} = d_{X,Y}^{-\alpha}$, with $X \in \{S, R_a, A\}$, $Y \in \{R_a, D, B\}$, and $\alpha$ being the path-loss exponent, which is set to 4.

Under the above assumptions, Fig. 2 shows the outage probability versus the transmit SNR, $\gamma_{P}$, for $N = 3$ and different values of $\gamma_{A}$ e $\gamma_{I}$. In this figure, it can be contrasted the effects over the outage performance of the interference temperature $\gamma_{I}$ constraint and the interference from the primary transmitter. First of all, it can be observed an excellent match between the simulation results and our exact expression for the outage probability. Also, it can be observed that our asymptotic expression also matches with the saturation limits on the system introduced by the different values for $\gamma_{I}$, thus confirming a diversity equal to zero for high SNR values, while the interference from the primary transmitter does not influence on the behavior of these limits, albeit significantly degrades the attained performance. In this sense, the higher the values of $\gamma_{A}$, the greater the performance loss, as can be observed in the curves for $\gamma_{I} = 10$ dB.

In Fig. 3 is depicted the outage probability versus the normalized distance between $S$ and $R_a$, $d_{S,R_a}/d_{S,D}$, for $\gamma_P = 15$ dB and $N = 3$. It can be observed that, as $\gamma_{A}$ decreases, the optimal relay position is located closer to $D$. This is due to the fact that, as the value of $\gamma_{A}$ increases, the first hop of the relaying link requires to be stronger in order to
\[
F_{\gamma_{Rn}}(\tau) = e^{-\frac{\gamma_{I}}{10^{\frac{SNR_n}{10}}}} + \frac{\gamma_{I}^{\frac{\gamma_{SRn}}{10^{\frac{\gamma_{SRn}}{10}}}}}{\gamma_{A}^{\frac{\gamma_{ARn}}{10^{\frac{\gamma_{ARn}}{10}}}}} Ei\left(\frac{-\gamma_{I}^{\frac{\gamma_{SRn}}{10^{\frac{\gamma_{SRn}}{10}}}}}{\gamma_{A}^{\frac{\gamma_{ARn}}{10^{\frac{\gamma_{ARn}}{10}}}}}ight)
+ \left(1 - e^{-\gamma_{SRn}}\right) \frac{\gamma_{P}^{\frac{\gamma_{SRn}}{10^{\frac{\gamma_{SRn}}{10}}}}}{\gamma_{A}^{\frac{\gamma_{ARn}}{10^{\frac{\gamma_{ARn}}{10}}}}} \left(1 + 1\right) - 1
\]

\[
P_{\text{OUT}} \approx 1 - \frac{\gamma_{I}^{\frac{\gamma_{Rn,s}}{10^{\frac{\gamma_{Rn,s}}{10}}}}}{\gamma_{A}^{\frac{\gamma_{ARn}}{10^{\frac{\gamma_{ARn}}{10}}}}} Ei\left(\frac{-\gamma_{I}^{\frac{\gamma_{Rn,s}}{10^{\frac{\gamma_{Rn,s}}{10}}}}}{\gamma_{A}^{\frac{\gamma_{ARn}}{10^{\frac{\gamma_{ARn}}{10}}}}}ight)
+ \left(1 - e^{-\gamma_{SRn}}\right) \frac{\gamma_{P}^{\frac{\gamma_{SRn}}{10^{\frac{\gamma_{SRn}}{10}}}}}{\gamma_{A}^{\frac{\gamma_{ARn}}{10^{\frac{\gamma_{ARn}}{10}}}}} \left(1 + 1\right) - 1
\]

Fig. 2. Outage probability vs. transmit SNR $\gamma_I$, for $N=3$, considering different values of $\gamma_A$ and $\gamma_I$.

Fig. 3. Outage probability vs. normalized distance, $d_{SRn}/d_{SD}$, for $N=3$, considering different values of $\gamma_A$ and $\gamma_I$.

V. CONCLUSIONS

In this paper, we have performed an exact analysis for the outage probability of an underlay cognitive relaying network with multiple relays, by considering partial relay selection and the effect of the interference coming from a primary user. Besides, an asymptotic closed-form expression is provided, which shows to be highly accurate for lower and higher values of SNR. Our expressions were validated via Monte Carlo simulations, whereby the primary-user interference over the secondary network was shown to cause a considerable loss in performance, specially for lower values of interference temperature, where the performance exhibits saturation limits. Additionally, the optimal relay position is observed to tend toward the destination as lower the value of the primary-transmitter SNR, while different values of interference temperature do not have effect over the outage performance, for a given transmit SNR at source and relays. Finally, in the considered system, the use of two relays produces a performance gain over the system, however, an increment on more than two relays does not entails any performance gain.
REFERENCES


