Tx-Rx Initial Access and Power Allocation for Uplink NOMA-mmWave Communications

Victoria Dala Pegorara Souto, Richard Demo Souza, Bartolomeu F. Uchôa-Filho

Abstract—This paper investigates the performance of non-orthogonal multiple access (NOMA) in a 2-user uplink millimeter-wave (mmWave) network. In particular, we consider a scenario where both the Base Station (BS) and the User Equipment (UE) are equipped with a uniform linear array (ULA). We consider beamforming both in the BS and the UE without any knowledge of the channel state information (CSI). As the power allocation intertwines with Initial Access (IA), in this paper, a joint IA and power allocation problem is considered to optimize the achievable sum rate with minimum rate constraint for each user. To solve this non-convex problem, we divide it into two optimization sub-problems (IA and Power Allocation), which are solved using Particle Swarm Optimization (PSO). Extensive simulations are conducted to compare the achievable sum rates of the NOMA and OMA systems. Results show that the proposed solution outperforms the OMA system.

Keywords—Initial Access, mmWave, Power Allocation, PSO.

I. INTRODUCTION

The fifth generation (5G) technologies are a new paradigm supported by the European Commission to overcome the challenges of the next generation networks, aiming to meet the new business requirements of vertical sectors like Industry 4.0, Smart Grids, and Smart Cities. In addition, millimeter-wave (mmWave) is a prospective technique for the 5G technology of wireless networks as it can provide much higher bandwidths, capacity, and data rate [1]. In addition, mmWave communications provide small wavelengths which allow for the use of antenna arrays formed by a large number of elements in the Base Station (BS) and/or in the User Equipment (UE). However, the use of beamforming at the BS and/or UE is challenging for Initial Access (IA) – the procedure that establishes a connection between UE and BS [2]. The IA is the main challenge of mmWave communications because the links generally require high directivity, which adds a long delay in the system. This delay must be reduced to meet the 5G requirement of end-to-end latency in the ms order for some applications [2].

Non-orthogonal multiple access (NOMA) is another potential technique for 5G and beyond wireless networks. Different from orthogonal multiple access (OMA), the idea of NOMA is to serve multiple users in the same resource block, where successive interference cancellation (SIC) is applied [3]. However, one challenge in NOMA is power allocation, which is crucial for the complete implementation of NOMA in the power domain [3]. Bringing these two techniques, mmWave and NOMA, together is the motivation of this work, aiming to maximize the system achievable sum rate.

A. Related Work

The application of NOMA in wireless networks is relatively new [4]. The main challenge faced is to solve the power allocation problem. For example, in [5] an uplink power control scheme is proposed aiming to maximize the outage performance and the achievable sum data rate for a NOMA uplink system. The power allocation and user clustering problems are investigated in [6], aiming to maximize the sum-throughput under transmission power constraints, minimum rate requirements of the users, and SIC constraints. Both the downlink and uplink scenarios were considered.

In addition, some works have studied the union of NOMA with mmWave communications, whose challenge is to solve a joint beamforming and power allocation problems. For example, in [7] the performance of downlink mmWave NOMA was evaluated considering random beamforming with fixed power allocation for a large number of users. In [8] joint beamforming and power allocation for the 2-user downlink mmWave NOMA scenario is investigated, aiming to maximize the achievable sum rate with minimum rate constraints for each user. As an expansion of [8], in [9] the authors consider the power allocation and beamforming problems jointly for a 2-user uplink NOMA mmWave scenario. While [8] and [9] consider only two UEs with a single antenna, in [10] this problem is investigated considering that both the UEs and BS are equipped with an analog phased array antenna.

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Palavras-Chave—Acesso Inicial, Ondas Milimétricas, Alocação de Potência, PSO.
The works in [5], [7]–[10] consider perfect channel state information (CSI) at the BS and UEs. However, CSI cannot be easily obtained in mmWave networks [2], [11], being the main challenge for the application of NOMA together with mmWave communications. This challenge motivates us to investigate a 2-user mmWave NOMA scenario, aiming to maximize the system sum rate, without any prior knowledge of the CSI in both the BS and the UE before the IA process.

In [12] we consider this problem for a 2-user uplink NOMA-mmWave system, but considering beamforming only at the BS, while the UE is equipped with a single antenna. However, with mmWave communications it is reasonable to assume a multiple antenna UE aiming to increase the beamforming gain at the IA, and consequently the user’s SNR. In spite of that, the use of multiple antennas at the users increase the delay at the IA process, and consequently the complexity of the optimization problem. Therefore, in this work we propose an extension of [12] to the case of multiple antenna UEs. The IA and power allocation problems are solved using Particle Swarm Optimization (PSO). Extensive computer simulations are conducted to compare the achievable sum rate of the proposed NOMA solution to that of conventional OMA.

### B. Contributions

The present work proposes a solution for joint IA and power allocation to maximize the achievable sum rate with minimum rate constraints while considering beamforming at the BS and UE without prior CSI. The contributions of this work are:

- The use of PSO in the joint IA and power allocation problem in mmWave NOMA systems considering beamforming at the BS and UE without prior CSI. As far as we know, all papers on this problem consider beamforming at the BS only, or suppose some previous CSI.
- The evaluation of the proposed solution with regard to achievable sum rate, varying the total transmit power, rate constraint, and number of antennas. Results show that the proposed solution can outperform OMA, more specifically Time Division Multiple Access (TDMA), in a 2-user uplink mmWave NOMA scenario.

### II. SYSTEM MODEL AND PROBLEM FORMULATION

#### A. System Model

We consider the 2-user uplink mmWave NOMA system, with \( N \) antennas in the BS and two users with \( M \) antennas each, i.e., there are beamforming at the BS and UE and both the BS and the UEs are equipped with a uniform antenna array (ULA). We do not consider any prior CSI at the BS or UEs.

![Block diagram of the complete problem solved in this work.](image)

As described next, in this work IA and power allocation are solved sequentially. The BS then applies NOMA decoding.

The whole process is illustrated in Figure 1. In practice the BS, from the appropriate control signals, knows that there are users in the cell which want to establish a physical link, and the IA process is initialized. Once the IA process for all users is successfully completed, the users begin to transmit uplink signals \( s_1 \) and \( s_2 \), at the same time and in the same frequency band, and the BS receives the combined signal

\[
Y_{\text{NOMA}} = u_1^H H_1 w \sqrt{p_1} s_1 + u_2^H H_2 w \sqrt{p_2} s_2 + zw^H,
\]

where \( u_k \in \mathbb{C}^{M \times 1} \) is the beamforming vector of user \( k \), \( w \in \mathbb{C}^{N \times 1} \) is the beamforming vector of the BS, \( H_k \in \mathbb{C}^{M \times N} \) is the channel between User \( k \) and the BS, \( z \in \mathbb{C}^{M \times 1} \) denotes the independent and identically distributed (IID) Gaussian white noise vector with power \( \sigma^2 \), \( E(\|s_k\|^2) = 1 \), and \( p_1 \) and \( p_2 \) are the transmit powers of Users 1 and 2, respectively. Each element of the beamforming vector \( w \in \mathbb{C}^{N \times 1} \) has a constant modulus (CM), i.e., \( |w_m| = \frac{1}{\sqrt{N}} \), \( m = 1, \ldots, N \) and each element of the beamforming vector \( u_k \in \mathbb{C}^{M \times 1} \) for \( k \in \{1, 2\} \) has a CM \( |u_{kl}| = \frac{1}{\sqrt{M}} \), \( m = 1, \ldots, M \) [8], [9].

We assume the Saleh-Valenzuela extended geometric channel model [13], often used in mmWave scenarios [14], [15], including \( N_c \) grouped propagation paths in \( N_c \) clusters, each cluster corresponding to a spreading path at a macro level and each path or subcluster being composed of several subpaths.

Thus, the channel between user \( k \) and the BS is

\[
H_k = \sqrt{\frac{NM}{N_c N_k}} \sum_{i=1}^{N_c} \sum_{j=1}^{N_k} \beta_{i,j} a_k(\phi_{i,j}) a_k^H(\theta_{i,j}),
\]

where \( \beta_{i,j} \) is the small-scale complex gain of the \( i \)-th subpath in the \( j \)-th cluster, \( a_k(\cdot) \) denotes the normalized transmit antenna responses, expressed as a function of the azimuth angle of departure (AoD) \( \phi_{i,j} \), and \( a_k(\cdot) \) denotes the normalized receive antenna responses for user \( k \), expressed as a function of the azimuth angles of arrival (AoA) \( \theta_{i,j} \). These vectors depend on the array geometry. Thus, for Uniform Linear Array (ULA), \( a_k(\cdot) \) and \( a_k(\cdot) \) are given by

\[
a_k(\phi) = \frac{1}{N} \left[ 1, e^{j \frac{2 \pi d}{\lambda} \cos(\phi_1)}, \ldots, e^{j \frac{2 \pi d}{\lambda} \cos(\phi_M)} \right],
\]

\[
a_k(\theta) = \frac{1}{M} \left[ 1, e^{j \frac{2 \pi d}{\lambda} \cos(\theta_1)}, \ldots, e^{j \frac{2 \pi d}{\lambda} \cos(\theta_N)} \right],
\]

where \( d \) is the distance between the array elements and \( \lambda \) is the wavelength. Following previous literature [13], [15], throughout this work we adopt the channel model parameters given by: \( N_c = 5, N_k = 10, d = \lambda/2, \beta_{i,j} \sim CN(0, 1) \), and \( \phi_{i,j} = \theta_{i,j} \sim U(0, 2\pi) \).

In NOMA, SIC is used [3], [4]. The decoding order depends on the channel and beamforming gains. Without loss of generality, assuming that User 1 has a better channel condition, i.e. \( |u_1^H H_1 w|^2 > |u_2^H H_2 w|^2 \), the achievable rates are

\[
R_1 = \log_2 \left( 1 + \frac{|u_1^H H_1 w|^2 p_1}{|u_2^H H_2 w|^2 p_2 + \sigma^2} \right),
\]

\[
R_2 = \log_2 \left( 1 + \frac{|u_2^H H_2 w|^2 p_2}{\sigma^2} \right).
\]
Moreover, the achievable rates in an OMA system, with total transmit power \( P \), are

\[
R_{k}^{\text{OMA}} = \frac{1}{2} \log_2 \left( 1 + \frac{|u_k^H h_k w|^2 P}{\sigma^2} \right), \quad k = 1, 2. \tag{5}
\]

### B. Problem Formulation

Aiming to maximize the achievable sum rate of the NOMA mmWave systems, we investigate two problems: IA and Power Allocation. We consider that each user must respect a minimal rate constraint, and the BS does not have prior CSI. Due to this, the combination of these two problems makes the optimization very complicated to be solved directly. Such maximization problem can be formulated as [10]

Maximize \( R_1 + R_2 \)

Subject to

\[
R_1 \geq r_1, \quad R_2 \geq r_2
\]

\[
0 \leq (p_1, p_2) \leq P, \quad (p_1 + p_2) = P \tag{6}
\]

\[
|\mathbf{w}|_n = \frac{1}{\sqrt{N}}, \quad n = 1, \ldots, N
\]

\[
|\mathbf{u}_k|_m = \frac{1}{\sqrt{M}}, \quad m = 1, \ldots, M,
\]

where \( r_k \) denotes the minimal rate constraint for User \( k \), and \( P \) denotes the total transmit power.

As already mentioned, Problem (6) is complicated to solve directly. Alternatively, we divided this problem into two sub-problems. Thus, this complicated optimization problem can be solved in two steps. The first step is to solve the IA problem, aiming to maximize the sum beamforming gain with the CM and a total transmit power constraint. This optimization problem can be formulated as [10]

Maximize \( u_k^H h_k w_1^2 + u_k^H h_k w_2^2 \)

Subject to

\[
|u_k^H h_k w_1|^2 > c_k, \quad c_k = \frac{(2^P - 1)}{\sigma^2},
\]

\[
|\mathbf{w}|_n = \frac{1}{\sqrt{N}}, \quad n = 1, 2, \ldots, N
\]

\[
|\mathbf{u}_k|_m = \frac{1}{\sqrt{M}}, \quad m = 1, 2, \ldots, M
\]

The second step is to solve the power allocation problem. Here we consider the beamforming array at the BS and the UE, found in (7), to determine the NOMA decoding order, and we maximize the achievable sum rate with minimal rate constraints. The optimization problem is [10]

Maximize \( R_1 + R_2 \)

Subject to

\[
R_1 \geq r_1, \quad R_2 \geq r_2
\]

\[
0 \leq (p_1, p_2) \leq P, \quad (p_1 + p_2) = P \tag{8}
\]

### III. PROPOSED SOLUTION

The IA and the power allocation problems evaluated in this work are described as following:

- **IA Problem**: First, we solve the IA problem, whereby the users establish a physical link with the BS [16], [17]. In this process, the pair of beamformings at the BS and at the UEs are sorted and tested based on (7). Then, for each beamforming pair, the feedback of the users’ SNR are received at the BS. From this feedback, the suboptimal beamforming pair and users’ positions are defined. In this work, we consider that the BS and the UE are equipped with a ULA, *i.e.*, with beamforming at the BS and UEs. However, the complexity of the IA problem increases considerably, seem intractable to solve this problem using analytic solutions or an exhaustive search technique [12]. Thus, we propose a solution based on PSO as described in Algorithm 1. From the proposed solution the best beamforming arrays \((\mathbf{u}_{\text{best}}, \mathbf{w}_{\text{best}})\) and effective channel gain of each user \(|u_k^H h_k w|^2\) are obtained. These results are achieved aiming to maximize the sum of the effective channel gain of the two users.

- **Power Allocation Problem**: In the same way as in [12], here we consider the power allocation problem for a 2-user uplink NOMA mmWave system. However, in this work, we consider the effective channel gain of each user, which is determined by the beamforming array at the BS and the UE obtained after solving the IA problem. Then, we can determine the NOMA decoding order. To solve the power allocation problem aiming to maximize the achievable sum rate, we consider the method based on PSO fromn [12] and described in Algorithm 2. The suboptimal solution of the power allocation problem is given by \(\text{power}_{\text{best}} = [p_1, p_2]\), obtained from Algorithm 2.

### A. Particle Swarm Optimization

PSO is an optimization approach inspired by the social behavior of animals [18]. In PSO, the so-called “swarm” is formed by \(L\) particles. These particles move across the search space with varying velocity. Their movement depends on the particle’s experience \((\varphi_{\text{best}})\) and the swarm experience \((\varphi_{\text{best}})\). Those variables are obtained by evaluating the position of each particle through a fitness function. For each iteration, the velocity and position of each particle are updated based on [18]

\[
\mathbf{v}_i^{t+1} = \omega \mathbf{v}_i^{t} + \ell_1 \text{rand}() \left( [\varphi_{\text{best}}] - \mathbf{x}_i^{t} \right) + \ell_2 \text{rand}() \left( [\varphi_{\text{best}}] - [\mathbf{x}_i^{t}] \right), \tag{9}
\]

\[
\mathbf{x}_i^{t+1} = \mathbf{x}_i^{t} + \mathbf{v}_i^{t+1} \tag{10}
\]

where \(t\) is the current iteration of particle \(t\) \((t = 1, \ldots, N_{\text{iter}}, i = 1, \ldots, L)\), \(\mathbf{v}_i^t\) and \(\mathbf{x}_i^t\) represent the velocity and the position of particle \(i\) at iteration \(t\), respectively, \(\ell_1\) and \(\ell_2\) are the particle learning factors, which define the influence of individual and collective experience on the position of particles, respectively, while the random number generator function \(\text{rand}()\) returns a number between 0 and 1 with uniform distribution. Moreover, \(\omega = (\omega_{\text{max}} - \omega_{\text{min}})/N_{\text{iter}}\) is the inertia velocity weight where \(\omega_{\text{max}}\) and \(\omega_{\text{min}}\) are the maximum and minimum of \(\omega\).

### IV. SIMULATION RESULTS

This section presents simulation results to evaluate the performance of the proposed solution based on Algorithm 1 and Algorithm 2. We consider the average results based on
Algorithm 1: PSO for the IA problem

Input: Number of antennas at the BS: \( N \)
Number of antennas at each user: \( M \)
Number of particles in the swarm: \( L \)
Maximum number of iterations: \( N_{it} \)
Learning factors: \( \ell_1 \) and \( \ell_2 \)
Range of inertia weight: \( \omega_{\text{max}} \) and \( \omega_{\text{min}} \)
Maximum Velocity: \( v_{\text{max}} \)

Output: \( \mathbf{w}_{\text{best}}, \mathbf{u}_{\text{best},k} \)

1. Initialize position \( \mathbf{x}^{\text{BS}}_i = \mathbf{w}_i \) and velocity \( \mathbf{v}^{\text{BS}}_i \)
2. Initialize position \( \mathbf{x}^{\text{UE}}_{k,i} = \mathbf{u}_{k,i} \) and velocity \( \mathbf{v}^{\text{UE}}_{k,i} \)
3. Find the global (\( g^{\text{BS}}_{\text{best}}, g^{\text{UE}}_{\text{best}} \)) and local (\( p^{\text{BS}}_{\text{best}}, p^{\text{UE}}_{\text{best}} \)) best solutions
4. for \( t = 1 : N_{it} \) do
5.   Calculate \( \omega \)
6.   for \( i = 1 : L \) do
7.     for \( n = 1 : N \) do
8.       Update \( \mathbf{v}^{\text{BS}}_i \) based on (9)
9.       Update \( \mathbf{x}^{\text{BS}}_i \) based on (10)
10.      if \( \mathbf{x}^{\text{BS}}_i \neq \frac{1}{\sqrt{N}} \mathbf{v}^{\text{BS}}_i \) then
11.        \( \mathbf{x}^{\text{BS}}_i = \frac{1}{\sqrt{N}} \mathbf{v}^{\text{BS}}_i \)
12.      end
13.   end
14.   for \( k = 1 : M \) do
15.     for \( m = 1 : M \) do
16.       Update \( \mathbf{v}^{\text{UE}}_{k,i} \) based on (9)
17.       Update \( \mathbf{x}^{\text{UE}}_{k,i} \) based on (10)
18.      if \( \mathbf{x}^{\text{UE}}_{k,i} \neq \frac{1}{\sqrt{M}} \mathbf{v}^{\text{UE}}_{k,i} \) then
19.        \( \mathbf{x}^{\text{UE}}_{k,i} = \frac{1}{\sqrt{M}} \mathbf{v}^{\text{UE}}_{k,i} \)
20.      end
21.   end
22.   Obtain the fitness function (7)
23.   Update \( g^{\text{BS}}_{\text{best},i}, p^{\text{BS}}_{\text{best},i} \)
24. end
25. Update \( g^{\text{UE}}_{\text{best},k}, p^{\text{UE}}_{\text{best},k} \)
26. end
27. \( \mathbf{u}_{\text{best},k} = p^{\text{UE}}_{\text{best},k} \)
28. \( \mathbf{w}_{\text{best}} = p^{\text{BS}}_{\text{best}} \)
29. return \( \mathbf{u}_{\text{best},k}, \mathbf{w}_{\text{best}} \)

Algorithm 2: PSO for the power allocation problem

Input: Number of Users: \( K \)
Number of particle swarm: \( L \)
Maximum number of iterations: \( N_{it} \)
Learning factors: \( \ell_1 \) and \( \ell_2 \)
Range of inertia weight: \( \omega_{\text{max}} \) and \( \omega_{\text{min}} \)
Maximum Velocity: \( v_{\text{max}} \)
Optimum beamforming vectors: \( \mathbf{u}_{\text{best},k}, \mathbf{w}_{\text{best}} \)

Output: \( \mathbf{p}_{\text{best}} \)

1. Initialize position \( \mathbf{x}^{\text{UE}}_{k,i} = [p_{i,k}] \) and velocity \( \mathbf{v}^{\text{UE}}_{k,i} \)
2. Find global \( g_{\text{best}} \) and local \( p_{\text{best}} \) best solutions
3. for \( t = 1 : N_{it} \) do
4.   Calculate \( \omega \)
5.   for \( i = 1 : L \) do
6.     for \( k = 1 : K \) do
7.       Update \( [v_{i,k}] \) based on (9)
8.       if \( v_{i,k} > v_{\text{max}} \) then
9.         \( [v_{i,k}] = v_{\text{max}} \)
10.      end
11.     Update \( [x_{i,k}] \) based on (10)
12.     if \( x_{i,k} > P \) then
13.        \( [x_{i,k}] = P \)
14.     end
15.     if \( x_{i,k} < 0 \) then
16.        \( [x_{i,k}] = 0 \)
17.     end
18.     Obtain the fitness function (8)
19. end
20. Update \( p^{\text{UE}_{\text{best},i}} \)
21. end
22. Update \( g_{\text{best}} \)
23. end
24. \( \mathbf{p}_{\text{best}} = g_{\text{best}} \)
25. return \( \mathbf{p}_{\text{best}} \)

10^3 channel realizations, \( N_{it} = 10^3 \) iterations for each channel realization, \( L = 10 \) particles per iteration, \( \ell_1 = \ell_2 = 2\), \( \omega_{\text{max}} = 0.9 \), and \( \omega_{\text{min}} = 0.2 \). These parameters have been defined from extensive simulations and performance evaluations. For comparison purposes, we consider the results obtained for OMA mmWave systems, and the results in [12] without considering beamforming at the UE.

Figure 2 compares the achievable sum rate between the proposed solution for NOMA mmWave system, and the OMA mmWave system, as a function of the minimum rate constraint for a different number of antennas at the UE. We consider \( N = 32\), \( P = 100 \) mW, and \( \sigma^2 = 1 \) mW. It is possible to observe that the proposed solution presents a much better performance than OMA. In addition, increasing the number of antennas at the UE improves system performance.

Figure 3 compares the achievable sum rate between the
proposed solution for NOMA mmWave system and the OMA mmWave system. This figure shows an evaluation of the achievable sum rate with varying maximal transmit power-to-noise ratio for different number of antennas at the UE, \(N = 32\), and \(r_1 = r_2 = 3\) bps/Hz. The results are similar to Figure 2, i.e., the proposed solution presents a much better performance than the OMA mmWave system. In addition, it is possible to observe that for \(M = 32\) the proposed solution achieve the same performance as the OMA mmWave system, with a reduction of 10dB in the transmit power-to-noise ratio.

\[
P/\sigma^2 = \frac{P}{\sigma^2} = 32
\]

This figure shows a comparison between the NOMA and OMA mmWave systems. It illustrates how the proposed solution significantly outperforms the OMA system, achieving a much better performance. The figure also highlights the benefits of using multiple antennas at the UE increase considerably the system performance. From the results, it is possible to attend that the proposed NOMA mmWave system significantly outperforms a conventional OMA mmWave solution.

Figure 3. Achievable rate versus maximal transmit power-to-noise ratio \((P/\sigma^2)\) for \(r_1 = r_2 = 3\), \(N = 32\), \(M = 8, 16\), and \(P = 100\) mW.

Figure 4 shows the achievable sum rate results when the users have different minimum rate constraints, \(r_2 = 5\) bps/Hz and different numbers of antennas at the UE. The results are compared with OMA mmWave system performance, and the achievable sum rate obtained in [12]. Again, the proposed solution achieves a considerably better performance than the OMA mmWave system. Moreover, it is possible to attend two users with a high minimum rate constraint. From the results, it is possible to observe that to add multiple antennas at the UE increase considerably the system performance.

\[
\text{V. Conclusion}
\]

We investigated how to maximize the achievable sum rate of a 2-user uplink mmWave NOMA system, considering that the BS and the UEs are equipped with a ULA, applying beamforming at both ends, without any previous CSI knowledge. This problem has been divided into two sub-problems: IA and power allocation. We proposed two implementations of the PSO technique to solve both sub-problems. The results showed that the proposed NOMA mmWave system significantly outperforms a conventional OMA mmWave solution.

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\text{Referências}
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