Outage Performance of Buffer-State-Based Relay Selection in Underlay CR-NOMA Networks

Vignon Fidèle Adanvo, Samuel Mafra and Samuel Montejo-Sánchez

Abstract—In this paper, we study a buffer-state-based relay selection scheme in a underlay cognitive radio (CR) non-orthogonal multiple access (NOMA) network limited by the interference limit accepted by the primary. To model the state of the relays in relation to packet numbers, it was necessary to implement the Markov chain model. The results of the simulations performed show validity of the proposed scheme. More specifically, in different relay position scenarios analyzed, results show that the best configuration is when the relay is closer to the nodes of the NOMA network. Beyond that, proper selection of the power allocation factor produces a better performance in outage probability. Finally simulation results show that the size of the relay buffer can improve the scheme and achieve full diversity.

Keywords—Buffer-equipped relay selection scheme; Cognitive radio; Non-orthogonal multiple access; Outage probability.

I. INTRODUCTION

The rapid development in the field of wireless communication services have increased the demand for spectrum resources by both secondary users (SUs) and primary users (PUs), making cognitive radio (CR) technology the most promising solution to increase spectrum reuse [1]. The non-orthogonal multiple access (NOMA), on the other hand, is regarded as one of the enabling technologies to meet the requirements of the high rate, massive connectivity and low latency in 5G and Internet of Things (IoT) [2]. CR is able to fill in spectrum holes and serve its SUs without causing harmful interference to PUs, while NOMA uses superposition coding and successive interference cancellation (SIC) enabling users with significantly different channel conditions to share the same resource block. In addition, the cooperative communication (CC) enables distributed nodes in a wireless network to collaborate, so as to realize a form of space diversity to combat the detrimental effects of fading [3].

In [4], the authors present a CR-NOMA, which outperforms the conventional CR frameworks in terms of spectrum efficiency and massive connectivity. However, the study presented in [5] demonstrates that, different from non-cooperative scheme for the CR-NOMA system, the proposed CC scheme ensures that a diversity order of the number of SUs is achieved. The outage probability (OP) of cooperative NOMA systems has also been evaluated in [6]–[10]. In [6], it is considered a partial relay selection (RS) scheme, which is overcome by the two-stage RS strategy proposed in [7]. In [8] and [9], it is investigated the OP of cooperative underlay CR-NOMA networks in two different scenarios; in [8] (in [9]), the authors restrict the transmit power of the relay (source), while they consider that the source (relay) does not interfere with PU due its sufficient remoteness. The optimal power allocation (PA) coefficients that improve the system performance were also found in [8], [9]. Then [9] is extended in [10], where it is shown that the PA factor and the number of relays have great impacts on outage performance.

In [11], the authors proposed two models with internal memory, the max-max protocol and the hybrid model, which provide greater benefit and flexibility in RS. However, the use of the buffers in the relay leads to an increase in end-to-end delay, since the information that arrives at the relay must be stored before being conveniently relayed. The max-link scheme of [12] proposes a buffer-aided RS. Although this proposition has a significant delay, for a large buffer size, better gain diversity is achieved. The buffer-aided RS considering the buffer state is proposed in [13], which improves the delay caused by the system compared to the scheme of [12].

The contributions of this work are as follows:

- We reformulated the RS scheme based on the buffer state, presented in [13], to improves the system performance, in terms of OP.
- Contrary to [10], here, two relays were implemented, which were selected by the buffer state and the availability of the channel.
- We extend the schemes of [8] [9], considering that both the source and the relay are limited by the PU interference limit.
- We modeled by Markov chain, the states of the relays in relation to the packet numbers. While, we show how the right PA factor can improve the OP.

The rest of the paper is organized as follows: Section II describes the implemented system model. The link selection scheme is presented in Section III. While in Section IV, the performance of the proposed scheme is evaluated by numerical and analytical comparison in terms of OP. Finally, the conclusions are presented in Section V.
II. SYSTEM MODEL

We consider a downlink cooperative underlay CR-NOMA system with buffer-state-based RS scheme, as shown in Fig.1. Moreover, Fig.1 illustrates the different scenarios that are considered to evaluate the system performance. The system model considers a primary destination $D_p$, a single secondary source $S$, two secondary relays $R_k$, where $k \in \{1, 2\}$, each relay node $R_k$ is equipped with a data buffer of a finite size with $L$ capacity and finally two secondary destinations $D_q$ where $q \in \{1, 2\}$, all the nodes are equipped with a single antenna. We assume that there is no direct link between the source and the destination nodes. The relays operate in the half-duplex mode based on the decode-and-forward (DF) principle. All the network such that the interference can be seen as noise. The target transmission rate is $r$ [bps/Hz]. We consider two operation phases, the transmission phase ($S \rightarrow R_k$) and the retransmission phase ($R_k \rightarrow D_q$). The RS strategy is based on the relay buffer state, as in [13].

According to the underlying CR network scenario, the transmissions of the SU s are only allowed when the interference caused by $S$ or $R_k$ to $D_p$ meets the minimum level of interference established. So, the transmit power of $S$ and the selected relay node $R_k$ can be expressed respectively as:

$$P_S \leq \frac{I}{|h_{SD_p}|^2},$$

$$P_{R_k} \leq \frac{I}{|h_{R_k,D_p}|^2},$$

where $I$ is the limit of interference accepted by $D_p$.

A. SOURCE-RELAY PHASE

In the transmission phase, NOMA was not used and the information to each SU is sent in orthogonal channels in time, i.e., the time-division mode. The signal-to-noise ratio (SNR) corresponding to the channels is expressed as:

$$\gamma_{SR_k} = \frac{I |h_{SR_k}|^2}{|h_{SD_p}|^2 N_0}.$$  \hspace{1cm} (3)

So, the outage probability of link $S \rightarrow R_k$ is

$$O_{SR_k} = \mathcal{P}(\gamma_{SR_k} < \epsilon_S)$$

$$= \int_{0}^{\infty} \int_{0}^{\infty} \frac{\lambda_{SR_k} e^{-\lambda_{SR_k} x} e^{-\lambda_{SD_p} y}}{\lambda_{SR_k} + \lambda_{SD_p}} dxdy$$

$$= 1 - \frac{\lambda_{SR_k}}{\lambda_{SR_k} + \mu \lambda_{SD_p}},$$ \hspace{1cm} (4)

where $\mu = (\frac{\epsilon_S N_0}{r})$, $\epsilon_S = 2^{r_0} - 1$ and $N_0$ is the noise spectral density.

B. RELAY-DESTINATION NOMA-BASED PHASE

In the retransmission phase, NOMA was used and the signal sent from the selected relay can be written as:

$$x = \sqrt{a} P_{R_k} x_1 + \sqrt{(1-a)} P_{R_k} x_2,$$ \hspace{1cm} (5)

where $a$ is the power allocation factor with $0 \leq a \leq 1$. $x_1$ and $x_2$ denote an information for the corresponding destination $D_q$ with the power constraint of $E[|x_q|^2] = 1$. So, the information received at $D_q$ is:

$$y_q = h_{R_k,D_q} x + w_q,$$ \hspace{1cm} (6)

where $w_q$ indicates the AWGN component at $D_q$.

We assume that $D_2$ is relatively further from $R_k$ than $D_1$, then the link $R_k \rightarrow D_1$ is stronger than the channel $R_k \rightarrow D_2$. Thus, according to the NOMA principle, the selected relay

\footnote{The transmission rate must be doubled in each phase, since the relays operate in the half-duplex mode, $2 \times r_0$. In addition, in the transmission phase, the time-division mode requires that it must be multiplied by the number of destination user, two in our system model, $2 \times 2r_0$.}
allocate more transmit power to the far destination information. Consequently, the instantaneous signal-to-interference plus noise ratio (SINR) of $D_2$ perceived by $D_q$ is:

$$\gamma_{R_k D_q} = \frac{(1-a)I|h_{R_k D_q}|^2}{a|I|h_{R_k D_q}|^2 + N_0|h_{R_k D_q}|^2}. \quad (7)$$

Then, after the SIC process, $D_1$ detects its own signal, where its SNR is given by:

$$\gamma_{R_k D_1, D_1} = \frac{a|I|h_{R_k D_1}|^2}{N_0|h_{R_k D_1}|^2}. \quad (8)$$

Hence, the outage probability of link $R_k \to D_1$ is

$$O_{R_k D_1} = P \left((\gamma_{R_k D_1, D_1} < \epsilon_{R_k}) \cup (\gamma_{R_k D_1, D_1} < \epsilon_{R_k}) \right)$$

$$= \int_0^\infty \int_0^\infty \frac{-e^{-\frac{\lambda_{R_k D_1}}{\lambda_{R_k D_p}}} e^{-\frac{\lambda_{R_k D_p}}{\lambda_{R_k D_p}}} dxdy}{1 + \lambda_{R_k D_1} \cdot \lambda_{R_k D_p}}$$

$$= \frac{\epsilon_{R_k} N_0}{\kappa \lambda_{R_k D_p}} \left[\lambda_{R_k D_1} \cdot \lambda_{R_k D_p}\right]$$

$$\text{with} \ \kappa = \frac{\epsilon_{R_k} N_0}{\lambda_{R_k D_p}} \text{where} \ \theta_{R_k} \equiv \min\{a(1-a(1-\epsilon_{R_k}))\}$$

and $\epsilon_{R_k} = 2^{2\theta_{R_k}} - 1$. While the outage probability of link $R_k \to D_2$ is

$$O_{R_k D_2} = P \left((\gamma_{R_k D_2, D_2} < \epsilon_{R_k}) \cup (\gamma_{R_k D_2, D_2} < \epsilon_{R_k}) \right)$$

$$= \int_0^\infty \int_0^\infty \frac{-e^{-\frac{\lambda_{R_k D_2}}{\lambda_{R_k D_p}}} e^{-\frac{\lambda_{R_k D_p}}{\lambda_{R_k D_p}}} dxdy}{1 + \lambda_{R_k D_2} \cdot \lambda_{R_k D_p}}$$

$$= \frac{1}{1 + \frac{\lambda_{R_k D_2}}{\lambda_{R_k D_p}}}$$

$$\text{with} \ \varpi = \frac{\epsilon_{R_k} N_0}{1-a(1-\epsilon_{R_k})}. \quad (9)$$

In order to guarantee the quality of service (QoS) for both destinations, we evaluate the link $R_k \to D_q$ by the pair outage probability. The metric is defined as the probability that at least one user could not correctly decode its own signal, which can be written as

$$O_{R_k D_q} = 1 - (1 - O_{R_k D_1})(1 - O_{R_k D_2}). \quad (10)$$

### III. LINK SELECTION ALGORITHM

In this section, we describe the link selection scheme, which is a extension of scheme proposed in [13], here considering a more general scenario. At the beginning of each slotframe, $S$, $D_1$ and $D_2$ transmit short reference signals. According to these reference signals, each $R_k$ estimates its respective channel states, assuming a symmetric channel model for the retransmission phase. Based on this information together with the respective buffer states, each relay decides if it is able to transmit and/or receive at each time interval, following the logic described in the Table I. For example in the first case, it is observed that when the relay has no packets to transmit, if in addition the link $h_{SR_k}$ is outage, then the relay remains silent. Another illustrative example is the fifth case, where the relay decides to receive, despite the availability of both links, since it has only one or no packets in its buffer.

### TABLE I

<table>
<thead>
<tr>
<th>Cases</th>
<th>$L_k$</th>
<th>Link SNR$_k$</th>
<th>Link $R_k D_q$</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$L_k = 0$</td>
<td>Outage</td>
<td>Outage</td>
<td>Silence</td>
</tr>
<tr>
<td>2</td>
<td>$L_k = L$</td>
<td>Outage</td>
<td>Outage</td>
<td>Receive</td>
</tr>
<tr>
<td>3</td>
<td>$L_k &lt; L$</td>
<td>Not outage</td>
<td>Outage</td>
<td>Receive</td>
</tr>
<tr>
<td>4</td>
<td>$L_k &gt; 0$</td>
<td>Outage</td>
<td>Outage</td>
<td>Transmit</td>
</tr>
<tr>
<td>5</td>
<td>$L_k &gt; 0$</td>
<td>Not outage</td>
<td>Not outage</td>
<td>Transmit</td>
</tr>
</tbody>
</table>

### TABLE II

<table>
<thead>
<tr>
<th>Cases</th>
<th>$L_{\text{max}}$</th>
<th>$L_{\text{min}}$</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$L_{\text{max}} = L$</td>
<td>$L_{\text{min}} = 0$</td>
<td>RX</td>
</tr>
<tr>
<td>2</td>
<td>$L_{\text{max}} &lt; L$</td>
<td>$L_{\text{min}} &gt; 0$</td>
<td>RX</td>
</tr>
<tr>
<td>3</td>
<td>$L_{\text{max}} = 1$</td>
<td>$L_{\text{min}} &gt; 0$</td>
<td>RX</td>
</tr>
</tbody>
</table>

Denote $T$ and $R$ as the set that contain relays able to transmit and receive, respectively, $\phi$ denotes the empty set. In the next stage, it is selected the best relay according to each situation: i) when $|R = \phi, T \neq \phi\rangle$, the relay with more packets in buffer within the ready-to-transmit relay set it is selected to transmit; ii) when $|R \neq \phi, T = \phi\rangle$, the relay with less packets in buffer within the ready-to-receive relay set is selected to receive; iii) when $|R = \phi, T = \phi\rangle$ all available links are in outage, so no relay is select; finally when $|R \neq \phi, T \neq \phi\rangle$ the relay from $T$ with more packets in buffer (we denote $L_{\text{max}}$) and the relay from $R$ with less packets in buffer (we denote $L_{\text{min}}$) are selected in order to define the action to execute. Based on these elements, it decides whether to transmit or receive according to Table II. We denote TX and RX the decision to transmit and receive, respectively.

Now, we extend the algorithm of [13] for a more general scenario in which the relays are in different positions in the network, differently of the original algorithm, that consider the relays nodes at a same distance of all nodes. The main objective is to minimize the interference caused by SUs to $D_p$ and take advantage of the best available channel to transmit on the network. In the original algorithm of [13], when all relays have the same amount of data, decide to receive data and the corresponding links are available, the source sends with an equal probability to the corresponding relay $P = K$, here $K = 2$. Contrary to [13], here it is decided to send to the relay with best channel gain within the source - relay channels. The probability of choosing this channel is shown below:

$$P_{R_2} = P \left(|h_{SR_2}|^2 > |h_{SR_1}|^2\right)$$

$$= \int_0^\infty \int_0^\infty e^{-\frac{\lambda_{SR_2}}{\lambda_{SR_1}}} e^{-\frac{\lambda_{SR_1}}{\lambda_{SR_2}}} dydx$$

$$= \frac{\lambda_{SR_2}}{\lambda_{SR_2} + \lambda_{SR_1}}, \quad (12)$$

with $P_{R_1} = 1 - P_{R_2}$.

In [13], when all the relays have the same number of packet in buffer and the corresponding links are available, the relay selection is made assuming the same probability for all the considered relays. Contrary to [13], here it is selected the relay.
with the best ratio between $R_k \rightarrow D_f$ and $R_k \rightarrow D_p$ links, here $D_f$ it is the farthest destination from relay $k$. In order to maximize the link gain to the far destination and to minimize the interference to $D_p$. So, the probability of choosing the relay $R_1$ is:

$$P_{R_1}^{*} = \mathcal{P}\left(\frac{|h_{R_1}D_2|^2}{|h_{R_1}D_p|^2} > \frac{|h_{R_2}D_2|^2}{|h_{R_2}D_p|^2}\right). \quad (13)$$

If we call $f_1 = \frac{|h_{R_1}D_2|^2}{|h_{R_1}D_p|^2}$ and $f_2 = \frac{|h_{R_2}D_2|^2}{|h_{R_2}D_p|^2}$ then the PDF of $f_1$ is $F_1(x) = \frac{x}{\lambda_{R_1D_2}\lambda_{R_1D_p}}$ and the PDF of $f_2$ is $F_2(y) = \frac{y}{\lambda_{R_2D_2}\lambda_{R_2D_p}}$.

$$P_{R_1}^{*} = 1 - \int_{0}^{\infty} \int_{0}^{\infty} F_1(x)F_2(y)\, dxdy \quad (14)$$

$$= 1 - \frac{\lambda_{R_1D_2}\lambda_{R_1D_p} + \lambda_{R_1D_2}\lambda_{R_2D_p} - \lambda_{R_1D_2}\lambda_{R_2D_p} \log\left(\frac{\lambda_{R_1D_2}\lambda_{R_2D_p}}{\lambda_{R_1D_p}\lambda_{R_2D_p}}\right)}{1 - \frac{\lambda_{R_1D_2}\lambda_{R_2D_p}}{\lambda_{R_1D_p}\lambda_{R_2D_p}}},$$

for $\lambda_{R_1D_1} \neq \lambda_{R_2D_1}$ and $\lambda_{R_1D_2} \neq \lambda_{R_2D_2}$, while $P_{R_1}^{*} = 0.5$ for $\lambda_{R_1D_1} = \lambda_{R_2D_1}$ and $\lambda_{R_1D_2} = \lambda_{R_2D_2}$. Otherwise, the outage probability that select $R_2$ is given by: $P_{R_2}^{*} = 1 - P_{R_1}^{*}$.

A. STATE TRANSITION MATRIX OF THE MC

In this subsection, we derive the outage probability of the proposed scheme based on Markov Chain analyses. First, we define $A$ state transition matrix of Markov chain of data packets staked in each buffer, $\Psi_n$ is the set of available state for state $S_n$ and $\Psi_n$ is the set of available links of state $S_n$. Based in [13], the probabilities of transiting from the $S_n$ state to the state $S_m$ is described by:

$$A_{mn} = \sum_{\psi_n \in \Psi_n} \mathcal{P}(\psi_n) \mathcal{P}(S_n \rightarrow S_m|\psi_n). \quad (15)$$

The probabilities to remains in the same state is

$$A_{nn} = \prod_{i,j \in \Psi_n} P_{ij}, \quad (16)$$

where $\mathcal{P}(\psi_n) = \prod_{h_{ij} \in \psi_n} (1 - P_{ij}) \prod_{h_{ij} \not\in \psi_n} P_{ij}$ is the probability of the event that a subset $\psi_n$ of the available links $\Psi_n$ can successfully deliver a packet. The steady-state probabilities are given by [12]

$$\pi = (A - I + B)^{-1}b, \quad (17)$$

where $B$ is a $(L + 1)^K \times (L + 1)^K$ matrix with all elements to be one, $\pi = [\pi_1, \ldots, \pi_{(L+1)^K}]^T$, $b = [1, \ldots, 1]^T$, $I$ is the identity matrix. Finally, the average outage probability is

$$O = diag(A)\pi. \quad (18)$$

IV. NUMERICAL RESULTS

In this section we present the numerical results, in order to evaluate the performance of the proposed CR-NOMA scheme in terms of outage probability. We evaluate OP for the three particular scenarios presented in Fig. 1. In all cases, we consider the Scenario 1 (see Fig. 1) with two relays and two packet capacities in the buffer ($L = 2$), except when the opposite is said. In total, there are nine states of the Markov chain to represent the buffer states as shown in Table III. The state transition diagram for the particular scenario is shown in Fig. 2. We omit the state-transition matrix by out of space in the document. We consider the transmission rate $r_0 = 1$ [bps/Hz], noise spectral density $N_0 = 1$, the path loss exponent $\alpha = 4$ and power allocation factor $\alpha = 0.2$. The interference limit accepted by $D_p$ is $I = 10$ [dB]. In the start point all buffers are empty. All distances are normalized with reference to $d_{SD_1}$.

Fig. 3a shows outage probability as function of the interference limit accepted by $D_p$, while Fig. 4a shows OP as function of the power allocation factor, in both cases for different buffer capacities, $L = \{0, 1, 2, 3\}$. We consider the Max-Min RS scheme for the case without buffer, $L = 0$. Note that for the particular scenario, the performance in terms of outage probability improves with the increment at the buffer capacity and that for $L \geq 2$ the system performance converges towards the same OP. Similar conclusions can be obtained in Fig. 4a when the impact of the power allocation factor $\alpha$ is investigated. Note that, OP improves as $I$ increases, see Fig. 3a, while the optimal value of $\alpha$ is close to 0.1 for all capacities considered, see Fig. 4a.

Fig. 3b shows outage probability as function of the interference limit accepted by $D_p$, while Fig. 4b shows OP as function of the power allocation factor, according to the active relay(s) in the Scenario 1 (see Fig. 1). Note that, system performance improves when the active relay is in the middle of the distance $d_{SD_1}$, but a greater benefit is achieved in terms of OP when the number of active relays increases. As can be seen, there is
a match between the numerical and theoretical results, which shows the validity of the work carried out. From Fig. 4b, we can see that the optimum value is $a = 0.1$ for the scenario with both active relays, which allows to achieve an OP of 0.003.

Fig. 3c shows outage probability as function of the interference limit accepted by $D_p$, while Fig. 4c shows OP as function of the power allocation factor, for the three proposed scenarios. Note that when both relays are active, the use of buffers finds a greater benefit by approaching the destination nodes to ensure successful delivery in the retransmission phase. In addition, this fact improves the performance of the CR-NOMA scheme, increasing the probability of successful decoding. Note that, the PA factor decreases as $R_1$ approaches the destinations, see Fig. 4c.

V. CONCLUSIONS

In this article, we investigated a buffer-state-based relay selection scheme in underlay cognitive NOMA network. The selection is based in state of the buffers and in the information of the instantaneous channels by each relay to the far destination and to primary destination. The results showed that a better result is obtained for the scenario where one of the relays is closer to the NOMA networks. Moreover, a better system performance can be obtained with an appropriate choice of the capacity of buffer at each relay and of the power allocation factor in the retransmission phase.

As a future work, we intend to analyze the impact of packet delay in a more realistic scenario with QoS guarantees. Moreover, we intend to implement a semi-markov model to analyze the proposed scheme in scenarios where the full transition matrix does not exist or cannot be attained without excessive overheads.

REFERENCES