Cooperative Spectrum Sensing Based on Skewness Statistical Tests

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Abstract—Cognitive Radio Networks are a solution for the ever growing requirement of higher wireless data rates and massive user accommodation on the electromagnetic radio spectrum. Spectrum sensing is one of the most important characteristics of these networks. It enables the detection or not of a licensed user in a specific band, and thus it can allow others to operate in opportunistic access. This paper analyzes the detection performance of the skewness statistical test in terms of spectrum detection in a cooperative cognitive radio network. The extraction of statistical parameters from the distribution of a received licensed user signal enables the evaluation of the detection probability of the method. Through Monte-Carlo simulations, it is compared to the traditional energy detection method. The results indicate that skewness statistical test is more efficient than the energy detection, and also that the more users on the network, the better its performance.

Keywords—Cognitive Radio, Cooperative Spectrum Sensing, Skewness Statistical Test.

I. INTRODUCTION

With the continuous growth of mobile devices, specially in the last century, there has been a greater demand for wireless communications with higher data rates. However, due to the inherent relation between greater data rates and a wider occupancy of the electromagnetic frequency spectrum, it can be seen that the current scheme of static frequency allocation is no longer proper. The occupancy of radio spectrum on bands below 3GHz varies widely between 15% to 85% [1]. In this way, the concept of Cognitive Radio (CR) arised as an alternative to this problem. The goal is to enable an opportunistic usage of licensed frequency bands that are not heavily occupied by licensed users or even vacant.

In cognitive radio context, the so-called primary or licensed users (PU’s) are the ones who possess a license to operate in a predetermined frequency spectrum band. On the other hand, the secondary users (SU’s) do not have priority to access these bands. Thus, in order to properly access such bands, it must be able to adapt some of its parameters, such as topology, operating conditions, and the standards’ requirements [2],[3].

In this context, this paper proposes a cooperative spectrum sensing method to be used on a cognitive radio network, considering statistical tests. Diverse metrics have already been exploited on literature considering statistical tests, such as: mean value, variance, skewness, kurtosis, and other higher-order moments [4]. Combinations of these parameters have also been considered, such as the combination of skewness and kurtosis in the well-known Jarque-Bera test [5]. In this paper, precisely, skewness measurements are used by the cognitive users to sense the evaluated frequency bands. Monte-Carlo simulations with a different number of users on the network are evaluated and then compared with the traditional energy detection method.

The rest of this paper is organized as follows: Section II exploits the concept of spectrum sensing, and its most standard method; Section III introduces cooperative spectrum sensing and decision fusion; Section IV details the statistical goodness-of-fit testing, with focus on the skewness measure; in Section V the skewness statistic test is then modeled to perform spectrum sensing. Also, all simulation parameters and their results are presented. The conclusions are stated in Section VI.

II. SPECTRUM SENSING

Spectrum sensing is one of the most important concepts in cognitive radio. It can be defined as the way of detecting, or becoming aware of the existing primary users transmitting in a certain frequency band and in a limited geographic area. With this technique, the cognitive users can detect “holes” in the spectrum and opportunistically use these bands that occasionally might be free. However, when the transmission of the licensed user signal is detected, the secondary has to immediately stop its transmission to ensure the quality of service for the registered user, given its priority. The traditional way of modelling a sensing problem is through a binary hypothesis test [3]:

\[ y[n] = \begin{cases} w[n], & \text{if } H_0 \\ h x[n] + w[n], & \text{if } H_1 \end{cases} \]

in which \( h \) is the channel gain, if considered. In this work, only an AWGN (Additive White Gaussian Noise) is evaluated; \( H_0 \) is the available channel hypothesis, given that only the AWGN discrete signal, \( w[n] \sim N(0, \sigma^2) \) was detected; and \( H_1 \) is the occupied channel hypothesis, signaling that there is in fact a transmission of a discrete signal \( x[n] \) with AWGN noise present on the channel [6].

In this context, in order to quantify the spectrum detection, some probabilities become fundamental:

- Detection Probability \( (P_d) \) : The probability of choosing \( H_1 \) when \( H_1 \) is true, which means that there exists a signal and it is detected.
\[ P_d = \mathbb{P}(\text{SignalDetected} \mid \mathcal{H}_1) \tag{2} \]

- False Alarm Probability \((P_{fa})\): The probability of choosing \(\mathcal{H}_1\) when \(\mathcal{H}_0\) is true, which means that there is no signal being transmitted, however the channel is wrongly classified as occupied.

\[ P_{fa} = \mathbb{P}(\text{SignalDetected} \mid \mathcal{H}_0) \tag{3} \]

- Missed Detection Probability \((P_{md})\): The probability of choosing \(\mathcal{H}_0\) when \(\mathcal{H}_1\) is true, which means that the signal is not detected, although it is present.

\[ P_{md} = \mathbb{P}(\text{SignalNotDetected} \mid \mathcal{H}_1) = 1 - P_d \tag{4} \]

The optimal spectrum sensing method is seen when \(P_d\) is maximised while \(P_{fa}\) is minimized [7].

There exists many techniques of spectrum sensing on literature, being their first classification as Wideband or Narrowband, according to the size of the band that will be evaluated. Also, another very used classifier is with respect to the type of primary signal, if the characteristics of the signal to be identified are previously known (Non-blind Sensing), if the detector knows only an estimate of the noise variance (Semi-blind Sensing) or if there is not any information about the signal being transmitted (Blind Sensing).

The most common technique in literature is the energy detection (ED) method. Its main disadvantage is to require an appropriate signal to noise ratio (SNR); however, due to its simplicity, it is always considered as a reference. The method consists in evaluating the energy of received signal, and if it surpasses a given threshold, then the signal is considered to be present, that is [7],[15]:

\[ \Lambda = \frac{1}{N} \sum_{n=1}^{N} |y[n]|^2 \tag{5} \]

\[ P_d = \mathbb{P}(\Lambda > \lambda \mid \mathcal{H}_1) \tag{6} \]

\[ P_{fa} = \mathbb{P}(\Lambda > \lambda \mid \mathcal{H}_0) \tag{7} \]

in which \(\Lambda\) is the energy detector test statistic, \(N\) is the number of samples of the received signal and \(\lambda\) is a predefined threshold. The distribution of the test statistic when applied on \(y[n]\) can then be evaluated by [8],[9]:

\[ p_{\Lambda}(z) = \begin{cases} \frac{1}{2\pi\sigma^2} e^{-\frac{z^2}{2\sigma^2}}, & \text{if } \mathcal{H}_0 \\ \frac{1}{2\pi} \left( \frac{\Gamma(\frac{\nu}{2})}{\sqrt{\nu}} \right) e^{-\frac{z^2}{2\sigma^2}} I_{\frac{\nu-1}{2}} \left( \frac{\nu}{\sqrt{\nu}} \frac{z\sigma}{\sqrt{2}} \right), & \text{if } \mathcal{H}_1, \end{cases} \tag{8} \]

in which \(u = WT\) is defined as the time-bandwidth product, being \(W\) the bandwidth and \(T\) the observation time, also given by the Nyquist criteria as \(u = N/2\); \(\gamma = E_a|h|^2/\sigma^2\) is the SNR, with \(E_a\) being the power budget of the primary user. Also, \(I_{\nu}(x)\) is the modified Bessel function, and \(\Gamma(\cdot)\) is Euler’s gamma function, given by

\[ \Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \tag{9} \]

It can be seen that \(\Lambda \sim \chi^2_N\) if \(\mathcal{H}_0\) (a Chi-Square distribution) and \(\Lambda \sim \chi^2\) (a non-central Chi-Square distribution) with both \(N\) degrees of freedom if \(\mathcal{H}_1\) and thus the following solutions arise: [10]

\[ P_d = Q_{\frac{\lambda}{\sqrt{2}\sigma}}(\sqrt{2}\gamma) \tag{10} \]

\[ P_{fa} = \frac{\Gamma(\frac{N}{2}, \frac{\lambda}{\sqrt{2}\sigma})}{\Gamma(\frac{N}{2})} \tag{11} \]

in which \(Q_{m}(\cdot, \cdot)\) is the \(m^{th}\) order generalised Marcum-Q function and \(\Gamma(\cdot, \cdot)\) is the upper incomplete gamma function defined as:

\[ Q_m(z, b) = \int_b^{\infty} x^{m-1} e^{-\frac{z^2}{2} + \frac{x^2}{2}} \cdot I_{m-1}(zx) dx \tag{12} \]

\[ \Gamma(z, x) = \int_x^\infty t^{z-1} e^{-t} dt \tag{13} \]

### III. Cooperative Spectrum Sensing

From the individual schemes that we have so far described, each SU would perform spectrum sensing to detect the presence or not of a PU, and thus decide if they are able to transmit in a specific band. However, with this type of network, the hidden terminal problem arises, that corresponds to the fact that even though the PU is currently transmitting, the SU is shadowed, that is, not able to detect, due to intense multipath fading or inside buildings with high penetration loss. Although, if the information coming from multiple SU users can be considered before taking the final decision, the problem of the hidden terminal can be mitigated, due to the spatial diversity that comes from this configuration, defined as cooperative spectrum sensing [12].

Some difficulties that appear when considering a cooperative cognitive radio network are noise problems caused by the appearance of a second channel, the reporting channel (in which the individual decisions are sent to either another cognitive user), on a distributed scheme or to a base station (on a centralised fashion). Also, different from a wireless sensor network, the cognitive cooperative networks are much more geographically distributed, which can be seen to be either good for a wider area knowledge or bad due to more intense signal fading and shadowing [13].

In this work the configuration considered is the centralized cooperative spectrum sensing with decision fusion on the base station. Each cognitive user takes an individual decision and reports this decision on a different channel via a BPSK (binary phase shift keying) modulation to a fusion center. Several fusion rules can be done, such as the AND rule (the channel is only considered free if all the cognitive users report \(\mathcal{H}_0\), the OR rule (the channel is considered free if at least one secondary user reports \(\mathcal{H}_0\), and the majority rule, in which the channel is considered available if more than half the users report \(\mathcal{H}_0\). The last one is shown to have the best performance [14], so is the one adopted for the simulations.

The above described fusion is modelled as:

\[ D_i = \begin{cases} 0, & \text{if } \delta(y) = \mathcal{H}_0 \\ 1, & \text{if } \delta(y) = \mathcal{H}_1, \end{cases} \tag{14} \]

\[ Z = \sum_{i=1}^{U} D_i \begin{cases} < K, & \mathcal{H}_0 \\ \geq K, & \mathcal{H}_1, \end{cases} \tag{15} \]
In which, $D_i$ is the individual user decision, $\delta(y)$ is the decision rule, based on statistical testing (16) and (26), $U$ is the total number of cognitive users in the system, $K$ is the minimal number of users in order to consider the channel as occupied, which in this case is $U/2$ (Majority), and $Z$ is the final or global test statistic, used to reach the final decision, given by the fusion center.

IV. Statistical Tests

Despite the fact of being the easiest method to implement spectrum sensing, the energy detection usually presents low performance, due to its reduced capability of detecting low power noisy signals. As an alternative to tackle this problem, different techniques have been proposed to optimize the spectrum sensing. One of such techniques is the spectrum sensing based on statistical goodness-of-fit tests, which consists in extracting statistical parameters from the received signal and then evaluating if the extracted sample set matches a targeted probability density function (pdf). The decision model becomes [10]:

$$\delta(y) = \begin{cases} H_0, & \text{if samples adhere to the target pdf} \\ H_1, & \text{if samples do not adhere to the target pdf.} \end{cases}$$ (16)

As the targeted pdf and the statistical parameters from the given received signal, which has gone through a AWGN channel, are previously known, then

$$y[n] \sim \text{Gaussian}, \quad \text{if } H_0$$

$$y[n] \sim \text{Non-Gaussian}, \quad \text{if } H_1.$$ (17)

This test is usually obtained from the use of higher moments, such as skewness, kurtosis, and other even higher moments and their combinations [4]. Consequently, after the computation of the test, the SU’s can decide if the channel is indeed occupied or not [6], [11]. In this work, the authors evaluate the skewness statistical test.

A. Skewness Detection Method

The skewness, $S$, of a given real-valued random variable $X$ is defined as its third standardised central moment, calculated by [16]

$$S = \frac{\mu_3}{\sigma^3} = \frac{\mathbb{E}[(X - \mathbb{E}[X])^3]}{(\mathbb{E}[(X - \mathbb{E}[X])^2])^{\frac{3}{2}}}$$ (18)

in which $\mathbb{E}[\cdot]$ is the expected value operator.

This measure quantifies the asymmetry of $X$’s distribution. If its skewness is equal to 0, then the distribution is balanced around the center. For a normal distribution, as an example, $S = 0$. If the skewness is positive, then it means that there is a greater concentration of values on the left side of the curve (longer right tail). And if it is negative, the values are relatively more frequent on the right side of the curve (longer left tail).

For a limited number $N$ of realizations, $\{x_i : X \rightarrow x_i, i = 1, ..., N\}$, the traditional Fisher-Pearson coefficient of skewness [17], which is based on the method of moments, can be used to evaluate $X$’s skewness $sd$ by

$$g = \frac{\mu_3}{\sigma^3} = \frac{1}{N} \sum_{i=1}^{N} \frac{(x_i - \bar{x})^3}{\left(\sum_{i=1}^{N} (x_i - \bar{x})^2\right)^{\frac{3}{2}}},$$ (19)

being $\mu_3$, $\sigma$, and $\bar{x}$, respectively, the third central moment, the standard deviation and the mean estimators of the set.

V. Experimental Methodology and Results

In order to perform the detection method based on skewness statistical test, initially each user calculates the FFT of the received signal, $Y[k] = F(y[n])$. In the case $H_0$, then both the real and complex parts of the FFT will also follow i.i.d Gaussian distributions, from (17) and [5]. Thus, it can be concluded that the amplitude spectrum will follow a Rayleigh distribution

$$A[k] = \sqrt{\text{Re}(Y[k])^2 + \text{Im}(Y[k])^2} \sim \text{Rayleigh}(\sigma)$$ (20)

So, as the method is applied to the $H_0$ hypothesis, the outcome will be the skewness of a Rayleigh distribution, which can be calculated using (18)

$$S_{\text{Rayleigh}} = \frac{2\sqrt{\pi(\pi - 3)}}{(4 - \pi)^2} \approx 0.631$$ (21)

By using the CFAR (Constant False Alarm Rate) principle the main goal is, given a fixed very small value of $P_{fa}$, to find a threshold $\lambda$ that maximizes $P_d$, that is

$$\max_{\lambda} P_d(\lambda)$$ (22)

Thus, the Neyman-Pearson criterion [18] can be applied in order to achieve the optimal threshold [11]

$$\int_{-\infty}^{-\lambda_{opt}} f_{g|H_0}(x)dx + \int_{\lambda_{opt}}^{\infty} f_{g|H_0}(x)dx = P_{fa}.$$ (23)

Which when solved enables [19]

$$\lambda = \sigma Q^{-1}(P_{fa}) + \epsilon,$$ (24)

in which $\epsilon$ is the estimation error, which will tend asymptotically to zero and $Q^{-1}(\cdot)$ is the inverse Q function, given by:

$$Q^{-1}(x) = \sqrt{2} \cdot \text{erfc}^{-1}(2x)$$ (25)

In this way, the final decision rule for each cognitive user can be found, and the network can be simulated according to the following test:

$$\delta(y) = \begin{cases} H_0, & \text{if } g(A[k]) < \lambda = \lambda(P_{fa}) \\ H_1, & \text{if } g(A[k]) \geq \lambda = \lambda(P_{fa}) \end{cases},$$ (26)

Using the software MATLAB©, histograms for the skewness estimator $g$ applied on the received signal were evaluated in order to obtain its behaviour for both the two hypothesis $H_0$ and $H_1$, for critical values of SNR.

Also, a cooperative cognitive network with 1, 5, and 10 users was evaluated when using the skewness detection method for spectrum sensing with $P_{fa} = 0.01$, and then for a fixed SNR of -19 dB, and compared with the standard energy detection method. For each case, 1000 Monte-Carlo simulations
were performed. The PU transmits $N = 5000$ BPSK symbols with a carrier frequency of $f = 200$ MHz. The sensing channel is an AWGN channel, with a varying SNR from -20 dB up to 15 dB. All the secondary users receive the signal each with an AWGN channel, and then a FFT is performed. This is done since, in [4], a better performance is shown when using the FFT bins on the HOS statistical test. It is executed with different FFT sizes ranging from 1024 up to 32768. However, in order to prevent a bigger number of samples, which directly affects sensing time, the FFT size is kept with 2048 points and 75 frames. Finally, the above described skewness detection method is executed (26). After, its decision is sent via a BPSK modulation (14) through the reporting channel to the fusion center, which performs decision fusion according to the majority rule stated in (15), and arrives to the conclusion if the channel is available or not.

A. Results

Figure 1 shows the distribution of the skewness estimator $g$, applied when the PU is not present (only noise) and when it is present (noisy signal), for two SNR values: -19 dB and -16 dB. It is observed in Figure 1.a) that, due to the overlapping of both cases, it remains hard to establish a proper threshold that defines well the hypothesis decision region. However, in Figure 1.b), it can be seen that now both distributions are quite separated, and thus, even though there is still some overlapping, the threshold calculated in (24) will define well each region.

Figure 2 shows the detection probability with respect to the sensing channel’s SNR, when using the energy or skewness methods, for a network with 1, 5 or 10 users. It can be verified that the skewness method shows a better performance than the energy method, since it reaches $P_d = 1$ around SNR = -16 dB, while the energy method only reaches this value around SNR = 0 dB.

Additionally, it can be verified that the cooperative spectrum sensing performs better than the non-cooperative detection. When considering 10 users, the detection performance achieves the value of $P_d = 1$ for a lower SNR. Specifically, the 5-users cooperative strategy requires a higher SNR of around 1 dB when compared to 10-users cooperative’s detection. When comparing the 10-user cooperative skewness sensing performance with the non-cooperative detection (1 user), the cooperative detection with 10 users presents a gain of around 2 dB to reach a detection probability equal to one.

Figure 3 exhibits the ROC detection curves for the proposed skewness detector over AWGN sensing channels. This analysis is obtained by fixing the SNR = -19 dB and by varying the probability of false alarm, in order to observe the behaviour of the detection probability. The increase in performance with more cognitive users on the network is also seen, once that in order to obtain overall greater $P_d$’s, the networks with more than one user can operate in smaller $P_{fa}$’s, giving more reliability to the cognitive system.

VI. Conclusions

This article presented a comparison between a cooperative and a non-cooperative (only one SU) cognitive radio network, in an AWGN sensing channel when using the skewness statistical test and the energy detection methods. This comparison was conducted based on an analysis of the detection probability. The spectrum sensing scheme that presented the best performance was the 10-user network, when detecting the presence of a PU signal via the skewness method. These results demonstrate the viability of the cooperative spectrum sensing based on skewness statistical test to spectrum sensing and its applications.

As a continuation of this research, the effects of fading channels that were not evaluated in this article will be considered for simulating channels that are more consistent with reality. Also, models that take account of the geographic position of each spatially-spread cognitive user, such as stochastic geometry models, may also enhance the reality of the simulations, always approaching it to real-world conditions. Finally, other statistical tests can be evaluated to analyze its performance in spectrum sensing applications.

REFERENCES

Fig. 2: Detector Performances Through an AWGN Sensing Channel

Fig. 3: Detector ROC Performance Through an AWGN Sensing Channel


