Sinc Exponential ISI-Free Pulse with Better Performance

Cesar A. Azurdia-Meza[1], Ismael Soto[2], Shaharyar Kamal[3]

Abstract— **In this manuscript, the sinc exponential pulse (SEP) is evaluated at the transmitter and receiver side by using different metrics. The impulse response of the SEP is derived, analysed, and compared with the traditional RC pulse. The SEP is characterized by introducing two new design parameters, adding degrees of freedom to optimize the pulse for a certain transmission scheme and roll-off factor. The SEP is optimized via numerical simulations to diminish the bit error rate (BER) in the presence of symbol timing errors. Eye diagrams are used to visually evaluate the sensitivity of the communication system due to intersymbol interference (ISI). In general, the SEP outperforms other existing pulses in terms of BER, evaluated for different symbol timing-errors and roll-off factors.**

Keywords— **Bit error rate (BER), intersymbol interference (ISI), sinc exponential pulse (SEP), symbol-timing errors.**

I. INTRODUCTION

The design of intersymbol free (ISI-free) band limited signals was a problem considered by Nyquist [1], [2]. Nyquist first criterion (Nyquist-I) guarantees that a sequence of pulses, sampled at uniformly and optimum spaced instants, will be ISI-free. Additionally to the ISI-free requirement, pulse shaping filters must possess low sensitivity to timing errors. In practical receivers, the presence of timing sampling errors causes the actual sampling points to deviate from the optimal ones; hence, symbol timing jitter is produced and the bit error rate (BER) increases. Therefore, the tails of the impulse response must decay rapidly outside the pulse interval in order to eliminate the undesired effects of time sampling errors in the adjacent pulses [2]–[4].

To address the prior concerns, several Nyquist-I pulses have been proposed. The most popular ISI-free Nyquist-I pulse for distortion-less transmissions is the traditional raised cosine (RC) pulse [1], [2]. The RC pulse has been proposed by the 3rd Generation Partnership Project (3GPP) as the pulse shaping filter to be implemented at the user equipment (UE) and at the base station (BS), for transmission and reception [5], [6]. Besides the RC pulse, other Nyquist-I pulses with lower BER values and wider eye openings have been proposed. In [1], the authors proposed several families of Nyquist-I pulses by using a parametric approach, adding more degrees of freedom in the design of ISI-free pulses. Further, the proposed pulses in [1] subsume previous ISI-free pulses as special cases, such as the RC pulse, among others. In [7], a family of ISI-free polynomial pulses is derived, whereas in [8] new families of Nyquist-I pulses are proposed using a linear combination of two polynomial pulses. Other ISI-free linear combination pulses have also been proposed in [2] and [9]. A novel family of ISI-free pulses with senary piecewise polynomial frequency characteristic is proposed in [10], whereas in [11] a new parametric family of Nyquist ISI-free pulses was proposed, denoted as piecewise flipped-exponential (PFE) pulse. All of the previous families of ISI-free pulses are characterized by having additional variables to the roll-off factor; therefore, adding supplementary degrees of freedom in the design.

The main objective of this manuscript is to evaluate the performance of the sinc exponential pulse (SEP) in baseband digital communication systems. The SEP was originally derived and sub-optimized to enhance the performance of orthogonal frequency division multiplexing (OFDM) based systems [12]. SEP is the product between an exponential expression and the modified similar raised cosine (SRC) pulse. The SRC was initially derived in [1] and modified in [12]. The sub-optimum SEP was designed to reduce inter-carrier interference (ICI) power and peak-to-average power ratio (PAPR), which are considered the major weaknesses in OFDM-based systems. In this manuscript, the SEP will be optimized numerically to diminish the bit error rate (BER) in the presence of symbol-timing errors. The performance of the sub-optimum SEP will be evaluated and compared with other recently proposed pulses for various roll-off factors, and symbol timing errors. Further, the eye diagram of the SEP will be evaluated and compared with the traditional RC pulse.

II. ISI-FREE SINC EXPONENTIAL PULSE

Nyquist first criterion for distortion-less transmissions within a bandlimited channel is defined as follows in the time domain [1], [2]

$$h(nT) = \begin{cases} 1, & n = 0 \\ 0, & n = \pm1, \pm2, \pm3, \pm4, \ldots \end{cases}, \tag{1}$$

where $h(t)$ is the impulse response of the system, and T is the symbol period. Whereas in the frequency domain, the Fourier transform of (1) is given as follows

$$\frac{1}{T}\sum_{m=-\infty}^{m=+\infty} H\left(f + \frac{m}{T}\right) = 1, \tag{2}$$

where $H(f)$ is the Fourier transform of $h(t)$. The symbol repetition rate is given by $T = 1/2B$ for a bandwidth

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[1] Department of Electrical Engineering, Universidad de Chile, Santiago, [2] Department of Electrical Engineering, Universidad de Santiago de Chile, Santiago, [3] Department of Electronics and Radio Engineering, Kyung Hee University, Suwon, Republic of Korea. Corresponding Author E-mail[1]: cazurdia@ing.uchile.cl. This work was partially supported by Project FONDECYT Iniciación, Grant No. 11160517.

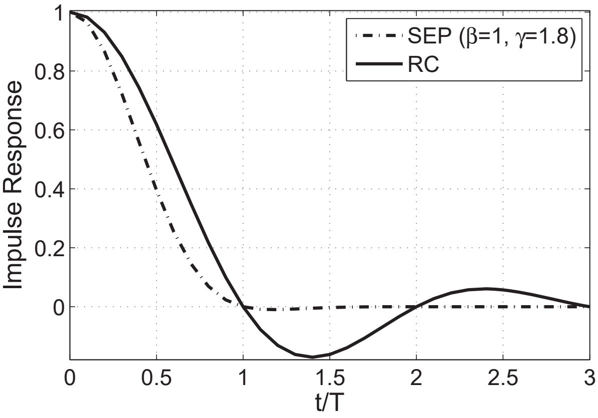

Fig. 1. Impulse response of the SEP and the RC pulse for an excess bandwidth $\alpha = 0.35$.

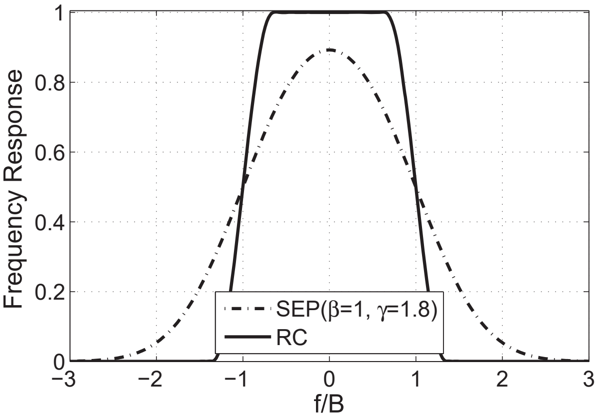

Fig. 2. Frequency response of the SEP and the RC pulse for an excess bandwidth $\alpha = 0.35$.

$B > 0$. Whereas the excess bandwidth of an ISI-free pulse is determined by the roll-off factor, defined for $0 \leq \alpha \leq 1$.

The SEP is characterized by a new design parameters. This new variable provides additional degrees of freedom in order to optimize the pulse for a certain task, technology, and roll-off factor. SEP is the product between an exponential expression and a modified version of the similar raised-cosine (SRC) pulse. The SRC was initially derived in [1] and modified in [12] by adding a new variable in the $sinc$ function as follows

$$h(t)_{SRC} = \frac{\sin(\beta t/T)}{(\beta t/T)} \times \frac{1 - 2\cos(\alpha\pi t/T)}{(3\alpha t/T)^2 - 1}, \tag{3}$$

where β is a new variable defined for all real numbers, while τ is the normalized time ($\tau = t/T$). The newly introduced β is used to control the phase of the $sinc$ function. The SRC pulse, given in (3), is multiplied with an exponential expression. The exponential expression introduces a new variable. The explicit time-domain expression of the SEP is given as follows

$$h(t)_{SEP} = e^{\left\{-\gamma(t/T)^2\right\}} \times h(t)_{SRC}, \tag{4}$$

where γ is a new design parameter defined for all real numbers. The variable γ provides an extra degree of freedom in the exponential expression. The family of pulses defined in (4), evaluated for $t = 0$, and for any value of α, β, and γ is always equal to one. Additionally, the SEP, evaluated for $n = \pm1, \pm2, \pm3, \pm4, \ldots$, and for any value of α, β, and γ, is always equal to zero. Therefore, the family of pulses described in (4) fulfills Nyquist's ISI-free criterion, previously described in (1). The even frequency-responde of the SEP can be described in terms of the parametric family of pulses defined in [1]. Throughout extensive numerical simulations, the sub-optimum SEP for BER reduction in the presence of time-sampling errors was determined for $\beta = 1$ and $\gamma = 1.8$. Therefore, in the rest of this manuscript we will evaluate the SEP using $\beta = 1$ and $\gamma = 1.8$. In general, there is an optimum β and γ for every roll-off factor and transmission scheme, although it might not be unique.

The impulse response of the SEP and the traditional RC pulse are plotted in Fig. 1 with a roll-off factor equal to 0.35, which is a roll-off factor commonly used in literature. It can be seen that the impulse response of the sub-optimum SEP has smaller relative magnitudes in its sidelobes compared to the traditional RC pulse. Further, it can be seen that the impulse response of the SEP decays rapidly. The trend is the same for other roll-off factors and values of β and γ. Consequently, robustness against ISI, a larger eye opening, and smaller PAPR are expected by implementing the SEP in single carrier systems [9], [13], [14]. The prior implies that the undesired effects of jitter will be diminished, and the sub-optimum SEP will be less sensitive to timing-sampling errors, resulting in a lower BER [1], [2], [7], [8].

The frequency response of the SEP and the traditional RC pulse are plotted in Fig. 2 for a roll-off factor equal to 0.35. It can bee seen from the frequency response that the SEP will introduce additional out-of-band radiation compared to the RC pulse. The trend is the same for other roll-off factors and values of β and γ. Therefore, a trade-off between BER reduction and out-of-band radiation exists.

III. PERFORMANCE EVALUATION

In this section the performance of the SEP is evaluated by using several practical tools. One of the tools used to analyse the performance of the SEP is the eye diagram. The eye diagram is a technique used to visually evaluate the susceptibility of the transmitted symbols to ISI. The eye diagrams were generated by superimposing 10^4 individual binary antipodal signalling sequences, and by inserting two consecutive symbol periods. Binary phase shift keying (BPSK) was the binary antipodal digital modulation used. In Fig. 3 we plotted the first 200 samples taken from the transmitted random antipodal signal, as well as the SEP pulse shape filtering. The eye diagram of the SEP and RC pulse with α equal to 0.35 are plotted in Figs. 4 and 5, respectively. The SEP exhibits a much wider eye opening than the RC pulse; therefore, a lower BER is expected for the SEP because it deals much better with timing jitter. The trend seen in Figs. 4 and 5 is the same for other roll-off factors and values of β and γ.

The last step of the evaluation process involves the calculation of the BER in the presence of time sampling errors

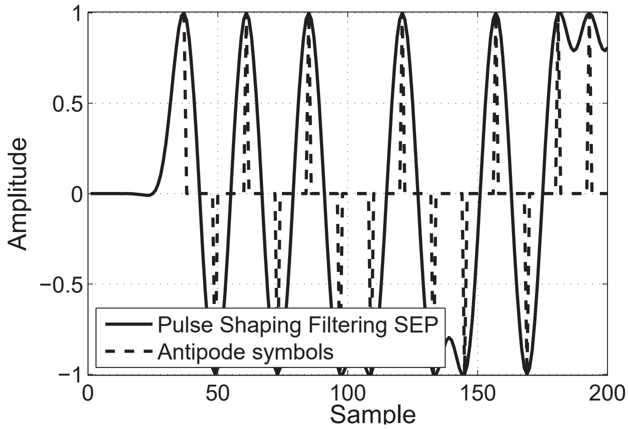

Fig. 3. SEP pulse shaping filtering for an excess bandwidth $\alpha = 0.35$.

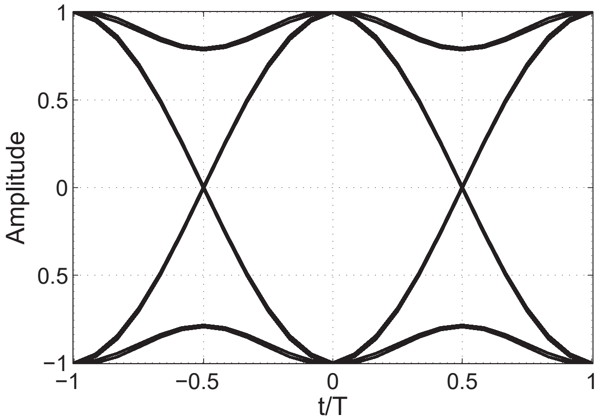

Fig. 4. Eye diagram of the SEP pulse for an excess bandwidth $\alpha = 0.35$.

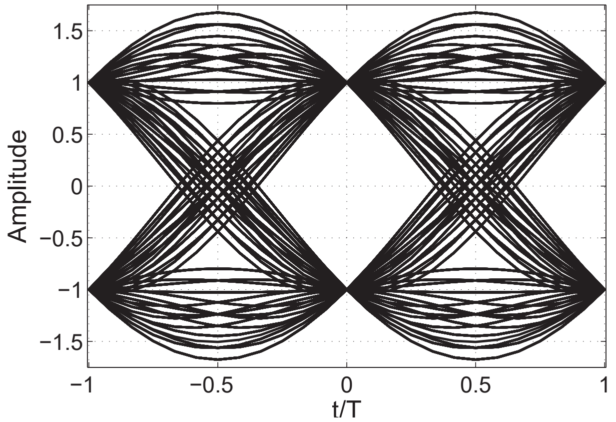

Fig. 5. Eye diagram of the RC pulse for an excess bandwidth $\alpha = 0.35$.

TABLE I

SYSTEM SIMULATION PARAMETERS

Parameter	Value
Symbol timing errors	$t/T = \pm 0.05, \pm 0.10, \pm 0.20$
Roll-off factor	$\alpha = 0.25, 0.35, 0.5$
M	100
T_f	1
Interfering Symbols	2^{10}
Channel	AWGN
Digital Modulation	BPSK
Signal-to-noise ratio	15 dB

and different roll-off factors. The BER is without a doubt the most important metric of performance because it considers the effects of noise, synchronization, and noise. To evaluate the BER of the sub-optimum SEP in the presence of time-sampling errors, the truncated Fourier series proposed in [15] is used. The method proposed in [15] is the de facto technique used to measure the BER of a Nyquist-I pulse in the presence of time-sampling errors. Let η represent the symbol timing error. If the probability density function of the symbol timing error is defined as $f(\eta)$, then the expected error probability due to ISI is defined as [15]

$$E\left[P_e\right] = \int P_e(\eta) f(\eta)\, d\eta, \tag{5}$$

where $P_e(\eta)$ is the error probability to ISI for a certain Nyquist-I pulse. For the case of additive white Gaussian noise (AWGN) in the channel and BPSK binary antipodal signalling, the error probability to ISI can be evaluated using the following truncated Fourier series [15]

$$P_e(\eta) = \frac{1}{2} - \frac{2}{\pi} \times \sum_{\substack{m=1\\ m=odd}}^{M} \left\{ \frac{\exp\left(-m^2 w^2/2\right)\sin(mwg_0)}{m} \right\} \prod_{\substack{k=N_1\\ k\neq 0}}^{N_2} \cos\left(mwg_k\right). \tag{6}$$

In (6), $w = 2\pi/T_f$ is the period used in the series, M represents the number of coefficients used in the truncated Fourier series, while N_1 and N_2 indicate the number of interfering symbols before and after the transmitted symbol, respectively. In the expression $g_k = p(kT + \eta)$, $p(t)$ is the ISI-free pulse evaluated. The parameters used to make the truncated Fourier series converge comply with the parameters used in previously reported manuscripts [3], [4], [8], [11]. These parameters are given in Table I. The BER in the presence of time-sampling errors was determined for the sub-optimum SEP, RC, as well as other recently proposed pulses. The families of Nyquist-I pulses proposed in [8] ($r(t)$, $q(t)$, and $v(t)$) are one of the newest families of pulses found in the literature. Further, the recently proposed PFE [11] is also evaluated, as well as the improved parametric linear combination pulse (IPLCP) [3], and the sinc parametric linear combination pulse (SPLCP) [4]. To the authors best knowledge, the SPLCP, IPLCP, and the PFE are the pulses with the best BER performance found in the literature. A signal-to-noise ratio (SNR) of 15 dB was used, while 2^{10} equally distributed interfering symbols (before and after the evaluated symbol) were generated.

The obtained results are tabulated in Table II. Note that in general the sub-optimum SEP has the smallest error rates, for different roll-off factors and timing offsets, compared to the RC, the pulses proposed in [8] ($r(t)$, $q(t)$, and $v(t)$), the PFE [11], as well as the IPLCP. This behaviour is consistent with the wider eye opening of the SEP. The SEP outperforms the SPLCP in terms of BER reduction for symbol timing errors $t/T = \pm 0.05, \pm 0.10$, but the SPLCP performs better than the

SEP for $t/T = \pm 0.20$. It can be seen that by increasing the value of the time-sampling error, for a fixed excess bandwidth α, the BER increases. Further, for a fixed sampling time error, a larger BER is obtained with a smaller roll-off factor.

TABLE II
BIT ERROR PROBABILITY FOR 2^{10} INTERFERING SYMBOLS AND SNR= $15dB$

α	Pulse	$t/T = 0.05$	$t/T = 0.10$	$t/T = 0.20$
0.25	RC	8.2189e-08	2.8184e-06	9.7472e-04
	r(t)	4.8482e-08	8.9851e-07	2.2072e-04
	q(t)	3.7570e-08	5.2618e-07	1.0717e-04
	v(t)	4.5112e-08	8.1936e-07	2.0823e-04
	IPLCP	1.5232e-08	5.8295e-08	3.7486e-06
	PFE	4.5110e-08	7.9603e-07	1.9140e-04
	SPLCP	1.3870e-08	4.4260e-08	1.9452e-06
	SEP	1.3277e-08	3.8132e-08	1.9425e-06
0.35	RC	6.0829e-08	1.3972e-06	3.9081e-04
	r(t)	3.2163e-08	3.7754e-07	6.7204e-05
	q(t)	3.0673e-08	3.5492e-07	6.5561e-05
	v(t)	3.1586e-08	3.5674e-07	6.1072e-05
	IPLCP	1.4476e-08	5.0372e-08	3.0138e-06
	PFE	3.0130e-08	3.3720e-08	5.6450e-05
	SPLCP	1.3380e-08	3.9432e-07	2.0438e-06
	SEP	1.3248e-08	3.7792e-08	1.9087e-06
0.50	RC	3.9728e-08	5.4893e-07	1.0022e-04
	r(t)	2.0147e-08	1.3248e-07	1.6136e-05
	q(t)	2.0718e-08	1.4907e-07	1.6818e-05
	v(t)	1.9166e-08	1.2175e-07	1.7891e-05
	IPLCP	1.3437e-08	3.9955e-08	2.0958e-06
	PFE	1.8921e-08	1.1615e-07	1.3072e-05
	SPLCP	1.2867e-08	3.4079e-08	1.5437e-06
	SEP	1.3208e-08	3.7288e-08	1.8538e-06

IV. CONCLUSIONS

In this manuscript the SEP was optimized numerically to diminish BER in the presence of time sampling errors. The evaluated pulse possesses a wider eye opening compared to the traditional RC pulse. The sub-optimum SEP pulse was determined for $\beta = 1$ and $\gamma = 1.8$. In general there is an optimum β and γ for every roll-off factor and transmission scheme, although it might not be unique. Overall the SEP generates smaller BER values compared to those of the other evaluated pulses. The latter was validated with different timing offsets and roll-off factors.

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REFERENCES

[1] N. C. Beaulieu and M. O. Damen, “Parametric construction of Nyquist-I pulses,” *IEEE Commun. Lett.*, vol. 52, no. 12, pp. 2134–2142, Dec. 2004.

[2] C. Azurdia-Meza, K. J. Lee, and K. S. Lee, “ISI-free linear combination pulses with better performance,” *IEICE Trans. Commun.*, vol. E96-B, no. 2, pp. 635–638, Feb. 2013.

[3] C. A. Azurdia-Meza, A. Falchetti, H. F. Arraño, S. Kamal, and K. Lee, “Evaluation of the improved parametric linear combination pulse in digital baseband communication systems,” in *International Conference on Information and Communication Technology Convergence (ICTC)*. IEEE, Oct. 2015, pp. 485–487.

[4] C. A. Azurdia-Meza, C. Estevez, A. D. Firoozabadi, and I. Soto, “Evaluation of the sinc parametric linear combination pulse in digital combination systems,” in *2016 8th IEEE Latin-Amerian Conference on Comunications (LATINCOM)*. IEEE, Nov. 2016.

[5] 3rd Generation Partnership Project (3GPP), “Universal Mobile Telecommunications System (UMTS); User equipment UE radio transmission and reception FDD,” European Telecommunications Standards Institute, Tech. Rep. 125.101, 2014.

[6] Third Generation Partnership Project (3GPP), “Universal Mobile Telecommunications System (UMTS); Base station BS radio transmission and reception FDD,” European Telecommunications Standards Institute, Tech. Rep. 125.104, 2014.

[7] S. Chandan, P. Sandeep, and A. K. Chaturvedi, “A family of ISI-free polynomial pulses,” *IEEE Commun. Lett.*, vol. 9, no. 6, pp. 496–498, June 2005.

[8] A. L. Balan and N. D. Alexandru, “Construction of new ISI-free pulses using a linear combination of two polynomial pulses,” *Telecommunication Systems*, vol. 59, no. 4, pp. 469–476, Aug. 2015.

[9] C. A. Azurdia-Meza, K. Lee, and K. Lee, “PAPR reduction in SC-FDMA by pulse shaping using parametric linear combination pulses,” *IEEE Communications Letters*, vol. 16, no. 12, pp. 2008–2011, Dec. 2012.

[10] A. L. Balan and N. D. Alexandru, “Improved Nyquist pulses produced by a filter with senary piece-wise polynomial frequency characteristic,” *Advances in Electrical and Computer Engineering*, vol. 14, no. 2, pp. 129–134, May 2014.

[11] N. D. Alexandru and A. L. Balan, “ISI-free pulse with piece-wise exponential frequency characteristic,” *AEU - International Journal of Electronics and Communications*, vol. 70, no. 8, pp. 1020–1027, Aug. 2016.

[12] S. Kamal, C. A. Azurdia-Meza, and K. S. Lee, “Improved Nyquist-I pulses to enhance the performance of OFDM-based systems,” *Wireless Pers. Commun*, 2017.

[13] B. Farhang-Boroujeny, “A square-root Nyquist M filter design for digital communication systems,” *IEEE Trans. Signal Process.*, vol. 56, no. 5, pp. 2127–2132, May 2008.

[14] S. Kamal, C. A. Azurdia-Meza, and K. S. Lee, “Low-PAPR hybrid filter for SC-FDMA,” *IEEE Commun. Lett.*, 2017.

[15] N. C. Beaulieu, “The evaluation of error probabilites for intersymbol and cochannel interference,” *IEEE Trans. Commun.*, vol. 39, no. 12, pp. 1740–1749, Dec. 1991.