On the Age-of-Information of Buffer-Aided Truncated ARQ Receiver with Chase Combining

Jorge Felipe Grybosi, João Luiz Rebelatto, and Guilherme Luiz Moritz

***Abstract*— In this work, we evaluate the average age-of-information (AoI) of a network composed of an internet-of-things (IoT) end-device (ED) transmitting randomly generated status updates to an access point (AP). More specifically, we elaborate on a recently proposed truncated automatic repeat request (TARQ) scheme, by considering a buffer-aided AP that is capable of combining retransmissions by means of the maximal ratio Chase Combining (CC) technique. While the average AoI of TARQ is a decreasing function of the generation probability p, we show that the proposed TARQ-CC scheme presents a more intricate relationship between its average AoI and p, which suggests that the generation probability can be optimized to minimize the AoI. Analytical and simulation results are presented, showing that TARQ-CC is capable of achieving considerably lower AoI than TARQ, without requiring any increase in the ED complexity.**

***Keywords*— Age-of-information (AoI), automatic repeat request (ARQ), chase combining, internet-of-things (IoT).**

I. INTRODUCTION

The accelerated expansion of internet-of-things (IoT) applications is mainly owed to low-power wide-area networks (LPWAN), which enables low-rate, long-range and low-energy consumption communications [1]. Among the leading representatives of large scale IoT market, one can mention Long Range (LoRa) and Narrowband-IoT (NB-IoT) [2], [3], which are adopted in applications such as weather reports, temperature, humidity, air quality in environmental monitoring [1], [2]. Such networks are commonly composed of end-devices (ED), which are responsible for monitoring dynamic systems and transmitting status updates to an access point (AP), where the timeliness of the updates is a feature that can both extend ED battery life and keep the information fresh at the AP.

The *Age-of-Information* (AoI) is a metric proposed to measure the freshness of the knowledge about the status of the ED at the AP [4], [5]. Formally, AoI is defined as the time elapsed since the last received status update generated by the ED. Conceptually, minimizing the AoI differs from classical throughput or delay optimizations [6]. By maximizing the throughput, one could expect real-time updates, but the backlog of multiple updates can insert a processing delay at the AP. Alternatively, minimizing the update rate can lead to outdated information.

Prior works as [4] evaluated the AoI under a first-come-first-serve (FCFS) queuing policy, but the results in [7] show that AoI in FCFS is mostly influenced by the server utilization, mainly because of backlog processing delays. The last-come-first-serve (LCFS) is studied in [7], and can achieve a lower AoI than FCFS. It is also shown that the preemption approach is better than non-preemption in terms of AoI. In a preemption-based scheme, the ED discards the current update in case a new (fresher) update becomes available, transmitting always the freshest update. Meanwhile, in a non-preemption scheme, the ED keeps transmitting the current update.

Automatic repeat request (ARQ) is a retransmission technique widely adopted to improve reliability, and whose benefits are also extended towards reducing AoI [8]–[11]. The ARQ with both FCFS and LCFS were compared in [8], under both preemption and non-preemption approaches, and it is shown that the average AoI can be reduced by keeping retransmitting the fresher status update indefinitely until the generation of a new update. Ceran *et al.* also studies retransmissions in [9], but assuming a feedback channel between AP and ED. Although transmitting unlimited replicas minimizes the average AoI, such approach may be very costly from an energy consumption perspective, limiting the life-time of battery-powered EDs. In this sense, an age-energy tradeoff (AET) is evaluated in [10], [11], by investigating the inherent AoI and energy consumption tradeoff of ARQ-based schemes. Specifically, [10] evaluates the AET of a feedback-aided network with retransmission, and [11] considered the scenario without feedback, proposing the so-called truncated ARQ (TARQ), which is a preemption scheme with a limited number of retransmissions. It is shown in [11] that the TARQ scheme achieves a much better AET than the scheme with unlimited retransmissions [8].

Regardless of the specific assumptions made in [8]–[11], a common fact is that the improvement in the average AoI comes mainly due to the reliability gain provided by retransmissions. In [11], however, the authors consider an approach where each retransmission is treated individually at the AP, *i.e.*, the receiver is not capable of combining independent replicas of a given update, which could improve reliability and consequently reduce the average AoI.

In this work, we extend the model from [11] by considering a buffer-aided AP, which is capable of combining independent replicas by means of maximal ratio Chase Combining (CC) [12]. Our results indicate that:

- The proposed TARQ-CC scheme achieves lower AoI levels than TARQ from [11], without increasing the complexity of the ED;
- While the AoI of TARQ is a decreasing function of the update generation probability p, we show that the AoI of TARQ-CC presents a more intricate relationship with p,

J.F. Grybosi, J.L. Rebelatto and G.L. Moritz, UTFPR, Curitiba-PR, Brazil (e-mail: jgrybosi@alunos.utfpr.edu.br, {jlrebelatto,moritz}@utfpr.edu.br)

This work has been partially supported by the National Council for Scientific and Technological Development (CNPq) and Coordination of Superior Level Staff Improvement (CAPES), Brazil.

presenting an optimal value that minimizes the AoI.

The rest of this paper is organized as follows: Section II presents the system model and some preliminaries. The performance of the proposed TARQ-CC scheme is evaluated in Section III. Section IV presents numerical results. Finally, Section V concludes the paper.

Notation: In this paper, $\mathbb{E}[\cdot]$ represents the expected value, $\mathcal{CN}(a, b)$ a complex normal distribution with average a and variance b, while $\Pr\{\phi\}$ is the probability of event ϕ.

II. Preliminaries

A. System Model

We consider a LPWAN composed of one end-device (ED) that sends status update information to an access point (AP). By considering that both ED and AP are provided with a single antenna and omitting the time index, the signal received at the AP after a transmission performed by the ED is

$$\mathbf{y} = \sqrt{P\,\kappa}\,h\,\mathbf{x} + \mathbf{w}, \quad (1)$$

where $\mathbf{x}$ is the unity-energy transmitted packet, P is the transmission power, κ accounts for the large scale path-loss and $\mathbf{w} \sim \mathcal{CN}(0, N_0)$ is the complex Additive White Gaussian Noise (AWGN), where N_0 is the thermal noise spectral density. The fading coefficient is modelled as $h \sim \mathcal{CN}(0, 1)$, *i.e.*, a Rayleigh block fading model, where the fading coefficient remains constant during one time-slot (TS) and changes in an independent and identically distributed (i.i.d) fashion between time-slots. The instantaneous signal-to-noise ratio (SNR) at the AP is then $\gamma = |h|^2\bar{\gamma}$, where $\bar{\gamma} = P\kappa/(N_0\,B)$ is the average SNR and B represents the channel bandwidth.

Following [11], we assume that at the beginning of each time slot a new status update is potentially generated at the ED according to a Bernoulli process, with generation probability p. The ED then keeps retransmitting such update to the AP until the occurrence of any of the following events: *i)* the maximum number of transmissions L is reached or *ii)* a fresher status update is available, which will then be transmitted.

B. Age-of-Information (AoI)

Let $r(t)$ represent the generation time of the most recently received status update, which is received at time slot t. The instantaneous AoI is then defined as the random process [11]

$$A(t) = t - r(t). \quad (2)$$

Fig. 1 illustrates the evolution of the instantaneous AoI from (2) in time, which, due to the discrete time-slotted model adopted in this work, presents a staircase shaping. Without loss of generality, we assume that $A(0) = 1$ in the example of Fig. 1. In the model, g_j is the generation time of the jth update, which, due to randomness of the fading, is not necessarily received by the AP. We also define t'_i as the time at which the ith update is received by the AP. The time between two consecutive successfully decoded updates is defined as $Y_i = t'_i - t'_{i-1}$, during which the AoI increases following the aforementioned staircase shaping, due to the absence of incoming updates. Upon the correct reception of the ith status update, the instantaneous AoI is reset to the AoI of such update, which we represent by $S_i = t'_i - g_j$, with $g_j = \max_j\{g_j | g_j < t'_i\}$. In Fig. 1, for example, $Y_2 = 5$ and $S_2 = 3$.

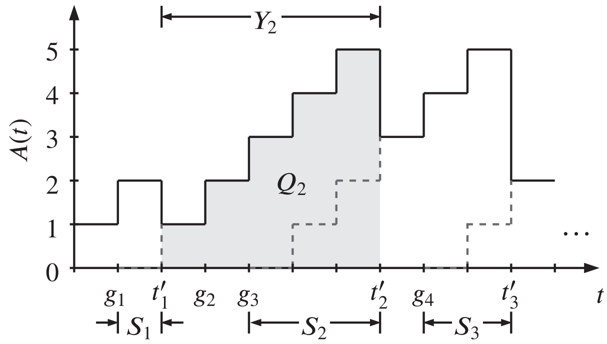


Fig. 1. Instantaneous AoI versus time slot index. Adapted from [11].

By defining $N_t = \max_i\{t'_i < t\}$ as the number of successfully received updates until time t and assuming ergodicity, the average AoI can be calculated using a sample average that converges to its corresponding stochastic average as [5], [11]

$$\bar{A} = \lim_{t\to\infty} \frac{1}{t}\sum_{i=1}^{N_t} Q_i = \frac{\mathbb{E}[Q_i]}{\mathbb{E}[Y_i]} = \mathbb{E}[S_{i-1}] + \frac{\mathbb{E}[Y_i^2]}{2\mathbb{E}[Y_i]} - \frac{1}{2}, \quad (3)$$

where Q_i is the area under $A(t)$, as highlighted in the gray area of Fig. 1 for $i = 2$, and $\mathbb{E}[\cdot]$ represents the expected value.

C. Truncated Automatic Repeat Request (TARQ)

The TARQ scheme from [11] adopts a truncated preemptive transmission approach, aiming at achieving an improved age-energy tradeoff when compared to a scheme with unlimited number of transmissions. The term *truncated* refers to a finite number of transmissions, while *preemptive* means that always the freshest status update is transmitted, regardless whether the previous (older) status update has been transmitted less than L times or not.

One important characteristic of TARQ is the fact that the AP treats each retransmission individually, *i.e.*, it does not have a buffer and cannot combine the signals received from previous transmissions. By adopting the geometric approach from (3), the average AoI of the TARQ scheme was shown to be [11, Eq. (33)]

$$\bar{A} = \frac{1 - q + pq}{(p - pq)\,(1 - (q - pq)^L)}, \quad (4)$$

where p is the probability that a new update is generated at the beginning of each time slot and q is the outage probability of a single transmission. Since [11] assumes that the AP treats each transmission individually, the outage probability for Rayleigh fading is given by

$$q = 1 - e^{-\lambda}, \quad (5)$$

where $\lambda = (2^R - 1)/\bar{\gamma}$, being R the attempted transmission rate (in bits per second per Hertz - bps/Hz). Note that the cases of a single update ($L = 1$) and with unlimited retransmissions ($L \to \infty$) from [8] are particular cases of (4).

Remark 1: The average AoI from (4) is a decreasing function of the update generation probability p, achieving its minimum value $\bar{A} = 1/(1-q)$ when $p = 1$, regardless the value of L.

As highlighted in Remark 1, since in the TARQ scheme each transmission is treated independently by the AP, the minimum average AoI is achieved when the ED generates and transmits a new status update per time-slot. In the extreme scenario with $p = 1$, the current freshest status update is always preempted by a new update in the next time-slot.

III. Buffer-Aided TARQ with Chase Combining

In this work, we propose the so-called TARQ-CC scheme by considering that the AP is provided with a buffer capable of storing up to L frames, being then able to store and combine the current transmission to its replicas previously received[1]. We adopt the Chase Combining (CC) scheme [12], which is a maximum ratio combining technique that maximizes the output SNR, and whose outage probability is obtained as [13] [14, Eq. (8.352.4)]

$$\begin{aligned} q_{\mathrm{CC}}(l) &= 1 - e^{-\lambda} \sum_{k=0}^{l-1} \frac{\lambda^k}{k!} \\ &= 1 - \frac{\Gamma(l,\lambda)}{\Gamma(l)}, \end{aligned} \tag{6}$$

where $1 \le l \le L$, being $\Gamma(l,\lambda)$ the upper incomplete gamma function, $\Gamma(l) = (l-1)!$ the complete gamma function, $l \in \mathbb{N}^*$.

Conjecture 1: In the preemptive truncated system model proposed in [11] and adopted in this work, the average AoI equals the expected value of the interdeparture time of two consecutive successfully decoded updates, *i.e.*

$$\bar{A} = \mathbb{E}[Y_i]. \tag{7}$$

We can infer (7) by noting that the average AoI from [11, Eq. (33)] equals the expected value of Y_i from [11, Eq. (24)]. Thus, upon resorting to Conjecture 1, the average AoI of the proposed TARQ-CC scheme is presented in the following Theorem.

Theorem 1: The average AoI of the proposed TARQ-CC scheme is given by:

$$\bar{A}_{\mathrm{CC}} = \frac{1-p}{p} + \frac{p\left(\Psi + \sum_{l=1}^{L} f(l)\, l\right) + f(L)\,(1-p)(L+1/p)}{p\left(1 - \sum_{l=1}^{L} f(l)\right) - f(L)\,(1-p)}, \tag{8}$$

where

$$f(l) = q_{\mathrm{CC}}(l)\,(1-p)^{l-1} p \tag{9a}$$

and

$$\Psi = e^{-\lambda p}\big(\Lambda + 1\big)\,\left(1 - q_{\mathrm{CC}}(L)\right) - \frac{e^{-\lambda}}{\Gamma(L)} \Lambda^L, \tag{9b}$$

being $\Lambda = \lambda(1-p)$.

Proof: Please refer to Appendix I. □

TABLE I

Parameters adopted in the numerical results [11].

Parameter	Value	Parameter	Value
Carrier frequency f_c	940 MHz	Bandwidth B	200 kHz
Noise density N_0	-174 dBm/Hz	Distance d	2000 m
Path loss κ	158 dB (Hata)	Data rate R	100 kbps/Hz

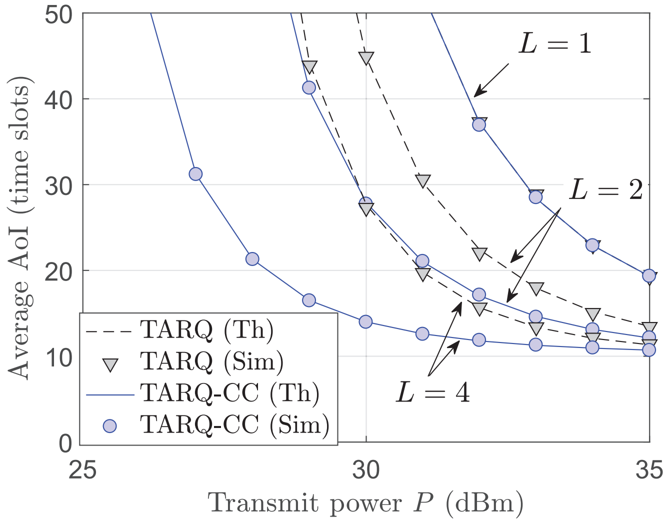


Fig. 2. Average AoI versus transmit power P, for $p = 0.2$ and maximum number of transmissions $L \in \{1, 2, 4\}$.

IV. Numerical Results

In this section, we evaluate the performance of the proposed TARQ-CC scheme by adopting the parameters of a standalone-based NB-IoT network, as presented in Table I and in accordance with [11]. In the figures, "Th" refers to theoretical (analytical) results, while "Sim" stands for simulation results, obtained by means of the Monte Carlo method.

The average AoI is presented in Fig. 2 as a function of the transmit power P, for $p = 0.2$ and $L \in \{1, 2, 4\}$. It can be seen that, as expected, the average AoI decreases with P for both schemes, due to the decrease in the outage probability. One can also see that TARQ-CC provides a considerable reduction in the AoI when compared to TARQ: while TARQ achieves an AoI of 27.6 time slots for $P = 30$ dBm and $L = 4$, TARQ-CC can achieve the same AoI, with the same transmit power, adopting only $L = 2$. Under the same transmit power $P = 30$ dBm, adopting TARQ-CC with $L = 4$ reduces the AoI to 14 time slots, an improvement of almost 50%. Finally, it is clear the agreement between the analytical and the simulated results, supporting Conjecture 1.

Fig. 3 evaluates the influence of the maximum number of transmissions L in the average AoI of both TARQ and TARQ-CC schemes, for $P = 28$ dBm and different update generation probabilities $p \in \{0.4, 0.6\}$. While the average AoI of both schemes decrease with L until reach a saturation, one can see that TARQ-CC achieves a reduced floor than that achieved by

[1] We consider that the AP is aware about whether the current transmission is a replica or a new update. Note that this information can be added to the message header.

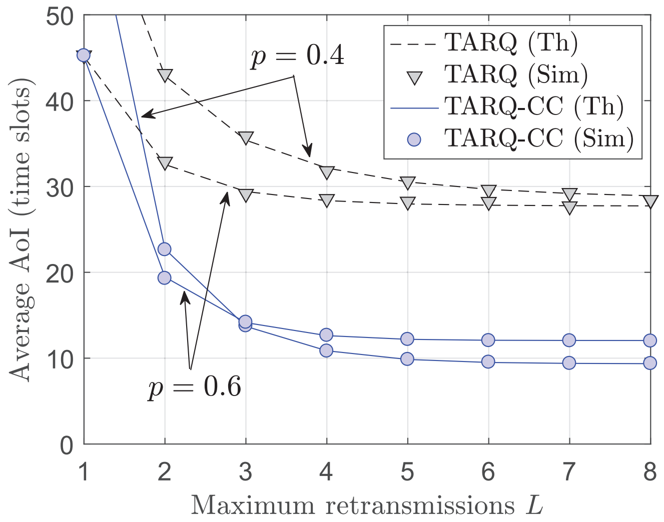


Fig. 3. Average AoI versus maximum number of transmissions L, for $P = 28$ dBm and update generation probability $p \in \{0.4, 0.6\}$.

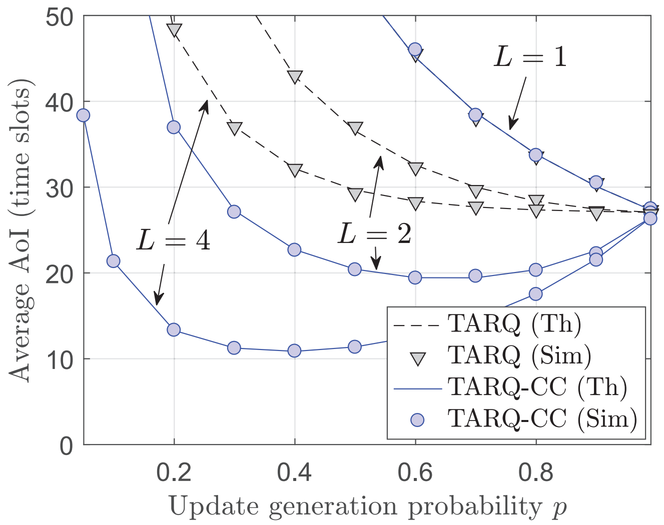


Fig. 4. Average AoI versus update generation probability p, for $P = 28$ dBm and maximum number of transmissions $L \in \{1, 2, 4\}$.

TARQ, for the same p. Moreover, an interesting observation is that, the higher the value of p, the lower the AoI floor of TARQ. However, a different outcome is observed to TARQ-CC, since the floor achieved by $p = 0.4$ is smaller than that achieved by $p = 0.6$. This suggests the existence of an optimal value of p that minimizes the average AoI of TARQ-CC.

Fig. 4 presents the average AoI versus the update generation probability p, for $P = 28$ dBm and $L \in \{1, 2, 4\}$. One can see that, as presented in Remark 1, the AoI of TARQ decreases with p, achieving the minimum value $\bar{A} = 1/(1-q)$ when $p = 1$, regardless the value of L. On the other hand, even though the TARQ-CC tends to the same value $1/(1-q)$ when p increases, it presents an optimal (lower) value of p that minimizes the average AoI and that varies with L. This can be explained by the fact that, with TARQ-CC, there is an intrinsic trade-off between increasing the generation probability and then transmit fresher updates more often, or retransmit a given (less fresh) update more times, which leads to a lower outage probability due to the combined decoding. In the scenario from Fig. 4, for example, the value of p that minimizes the average AoI is $p = 0.6$ and $p = 0.4$ for $L = 2$ and $L = 4$, respectively.

Finally, it is worthy mentioning that, even though the proposed scheme requires more advanced signal processing capabilities than TARQ, such increase in complexity is needed at the AP-side only, being transparent from the ED perspective.

V. Final Comments

In this work, we evaluated the average age-of-information (AoI) of a network composed of an IoT device transmitting randomly generated status updates to a buffer-aided access point (AP), operating under the Chase combining (CC) technique. Our results indicate that the proposed truncated automatic request (TARQ)-CC scheme can achieve lower levels of AoI than a recently proposed TARQ scheme. Moreover, we also showed that the AoI of the proposed scheme varies in a non-trivial fashion with the update generation probability. Future works include a more detailed analytical evaluation of the optimal update generation probability that minimizes the AoI of TARQ-CC.

Appendix I
Proof of Theorem 1

Assuming that Conjecture 1 is valid, we need to obtain the expected value of Y_i, the interdeparture time of two successfully decoded updates. Let W_i be the waiting time from the reception of the $(i-1)$th status update until G_i, the generation time of the next status update, and K_i be the time from G_i until an update being correctly received at the AP. One than has that:

$$\mathbb{E}[Y_i] = \mathbb{E}[W_i] + \mathbb{E}[K_i]. \tag{10}$$

Since the update generation follows a Bernoulli process with generation probability p, one has that $\Pr\{W_i = k\} = (1-p)^k p$, such that the expected value of W_i can be shown to be [14, Eq. (0.231.2)] [11, Eq. (12)]

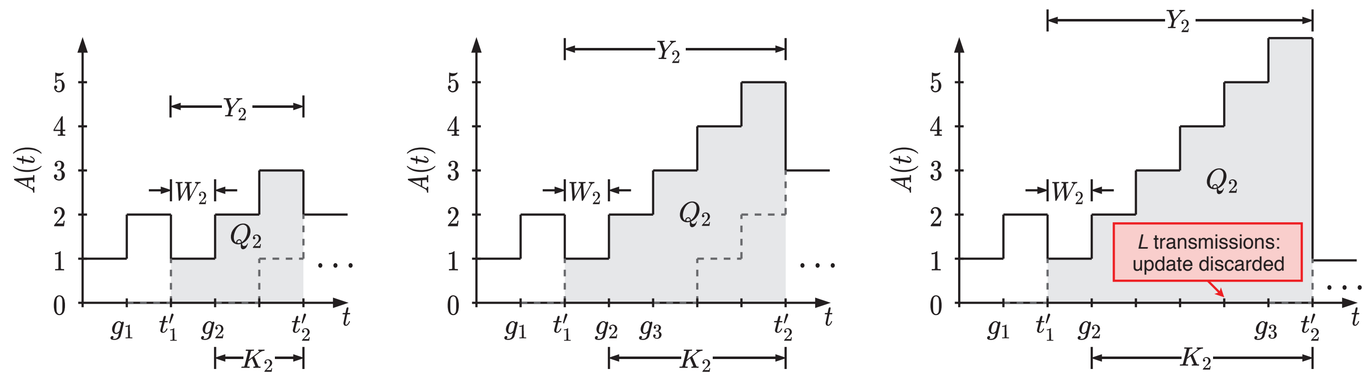
$$\mathbb{E}[W_i] = \sum_{k=1}^{\infty} \Pr\{W_i = k\} = \frac{1-p}{p}. \tag{11}$$

From the perspective of the first update generated after the correct decoding of the $(i-1)$th status update, there are three different outcomes that influence the value of K_i [11]:

Case 1. Such update is correctly decoded by the AP;
Case 2. It is preempted by one (or more) fresher updates;
Case 3. It is not correctly decoded after L transmissions, being discarded and making the ED to wait for a new update, which can potentially be preempted by other updates.

Such events are illustrated in Fig. 5. Thus, since these events have complementary probability, the expected value of K_i is

$$\mathbb{E}[K_i] = \sum_{a=1}^{3} \mathbb{E}[K_i^a], \tag{12}$$



(a) **Case 1** (b) **Case 2** (c) **Case 3**

Fig. 5. Example of the evolution of K_i, for $i = 2$ and considering $L = 3$. In (a) the update generated at time g_2 is correctly decoded without being preempted nor exceeding the maximum number of transmissions; In (b) it is preempted by a new update at time g_3; and in (c) the maximum number of allowed transmissions is exceeded and it is discarded, waiting for a new update.

where $\mathbb{E}[K_i^a] \stackrel{\text{def}}{=} \mathbb{E}[K_i|\textbf{Case } a]\Pr\{\textbf{Case } a\}$, with $a \in \{1,2,3\}$, being $\Pr\{\textbf{Case } a\}$ the probability of **Case** a and $\mathbb{E}[K_i|\textbf{Case } a]$ the correspondent conditional expected value.

When adopting CC, the probability that the AP correctly decode the update exactly with lth transmissions (with $l \leq L$) is given by:

$$d(l) = (1-p)^{l-1} q_{\text{CC}}(l-1)\left(1 - \frac{q_{\text{CC}}(l)}{q_{\text{CC}}(l-1)}\right), \quad (13)$$

where $(1-p)^{l-1}$ is the probability that no new update was generated in the previous $l-1$ time slots (there is no preemption), $q_{\text{CC}}(l-1)$ is the outage probability of a maximum ratio combining with $l-1$ branches as from (6) (meaning that the previous $l-1$ transmissions were not correctly decoded after CC) and $(1 - q_{\text{CC}}(l)/q_{\text{CC}}(l-1))$ is the probability that the update is correctly decoded in the lth transmission, given that it was not decoded in the $(l-1)$th transmission. The value of $\mathbb{E}[K_i^1]$ is then obtained as:

$$\begin{aligned}\mathbb{E}[K_i^1] &= \sum_{l=1}^{L} d(l)\, l \\ &= \sum_{l=1}^{L} l\,(1-p)^{l-1}\Big[q_{\text{CC}}(l-1) - q_{\text{CC}}(l)\Big] \quad (14)\\ &= e^{-\lambda p}\big(\Lambda+1\big)\big(1 - q_{\text{CC}}(L)\big) - \frac{e^{-\lambda}\Lambda^L}{(L-1)!},\end{aligned}$$

where the last equality comes with the aid of [14, Eq. 8.352.6] and $\Lambda \stackrel{\text{def}}{=} \lambda(1-p)$.

As mentioned in [11], Case 2 and Case 3 have the complicating factor that K_i can reach infinity due to multiple preemptions. The solution adopted in [11] (which is derived from [8]) considers a recursive method that leads to

$$\mathbb{E}[K_i^2] = \sum_{l=1}^{L-1} q_{\text{comb}}(l)\,(1-p)^{l-1} p\,(l + \mathbb{E}[K_i]) \quad (15)$$

and

$$\mathbb{E}[K_i^3] = q_{\text{comb}}(L)\,(1-p)^{L-1}\,(L + \mathbb{E}[W_i] + \mathbb{E}[K_i]), \quad (16)$$

where $q_{\text{comb}}(l)$ is the outage probability of the combining scheme employed at the AP, after l transmissions. In the TARQ scheme from [11], one has that $q_{\text{comb}}(l) = q^l$, while in this work $q_{\text{comb}}(l) = q_{\text{CC}}(l)$. For a more detailed explanation about (15) and (16), we refer the reader to [11, Eq. (17)].

Upon replacing (11) in (16), placing (14), (15) and (16) in (12), isolating $\mathbb{E}[K_i]$ and placing it in (10) along with (11), one can finally obtain the average AoI of TARQ-CC as presented in (8), after a few mathematical manipulations, concluding the proof.

REFERENCES

[1] U. Raza, P. Kulkarni, and M. Sooriyabandara, "Low power wide area networks: An overview," *IEEE Commun. Surveys Tuts.*, vol. 19, no. 2, pp. 855–873, 2017.

[2] A. Ikpehai, B. Adebisi, K. M. Rabie, K. Anoh, R. E. Ande, M. Hammoudeh, H. Gacanin, and U. M. Mbanaso, "Low-Power Wide Area Network Technologies for Internet-of-Things: A Comparative Review," *IEEE Internet Things J.*, vol. 6, no. 2, pp. 2225–2240, April 2019.

[3] M. Chen, Y. Miao, Y. Hao, and K. Hwang, "Narrow band internet of things," *IEEE Access*, vol. 5, pp. 20 557–20 577, Sep. 2017.

[4] S. Kaul, R. Yates, and M. Gruteser, "Real-time status: How often should one update?" in *Proc. IEEE INFOCOM*, Mar. 2012, pp. 2731–2735.

[5] A. Kosta, N. Pappas, and V. Angelakis, "Age of information: A new concept, metric, and tool," *Foundations and Trends® in Networking*, vol. 12, no. 3, pp. 162–259, 2017.

[6] M. A. Abd-Elmagid and H. S. Dhillon, "Average peak age-of-information minimization in UAV-assisted IoT networks," *IEEE Trans. Veh. Technol.*, vol. 68, no. 2, pp. 2003–2008, Feb. 2019.

[7] S. K. Kaul, R. D. Yates, and M. Gruteser, "Status updates through queues," in *Proc. CISS*, Mar. 2012, pp. 1–6.

[8] K. Chen and L. Huang, "Age-of-information in the presence of error," in *Proc. IEEE ISIT*, Jul. 2016, pp. 2579–2583.

[9] E. T. Ceran, D. Gündüz, and A. György, "Average age of information with hybrid ARQ under a resource constraint," *IEEE Trans. Wireless Commun.*, vol. 18, no. 3, pp. 1900–1913, Mar. 2019.

[10] J. Gong, X. Chen, and X. Ma, "Energy-age tradeoff in status update communication systems with retransmission," in *Proc. IEEE GLOBECOM*, Dec. 2018, pp. 1–6.

[11] Y. Gu, H. Chen, Y. Zhou, Y. Li, and B. Vucetic, "Timely status update in internet of things monitoring systems: An age-energy tradeoff," *IEEE Internet Things J.*, vol. 6, no. 3, pp. 5324–5335, Jun. 2019.

[12] D. Chase, "Code combining - a maximum-likelihood decoding approach for combining an arbitrary number of noisy packets," *IEEE Trans. Commun.*, vol. 33, no. 5, pp. 385–393, May 1985.

[13] A. Goldsmith, *Wireless Communications*. New York, USA: Cambridge University Press, 2005.

[14] I. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. Academic Press - Elsevier, 2007.